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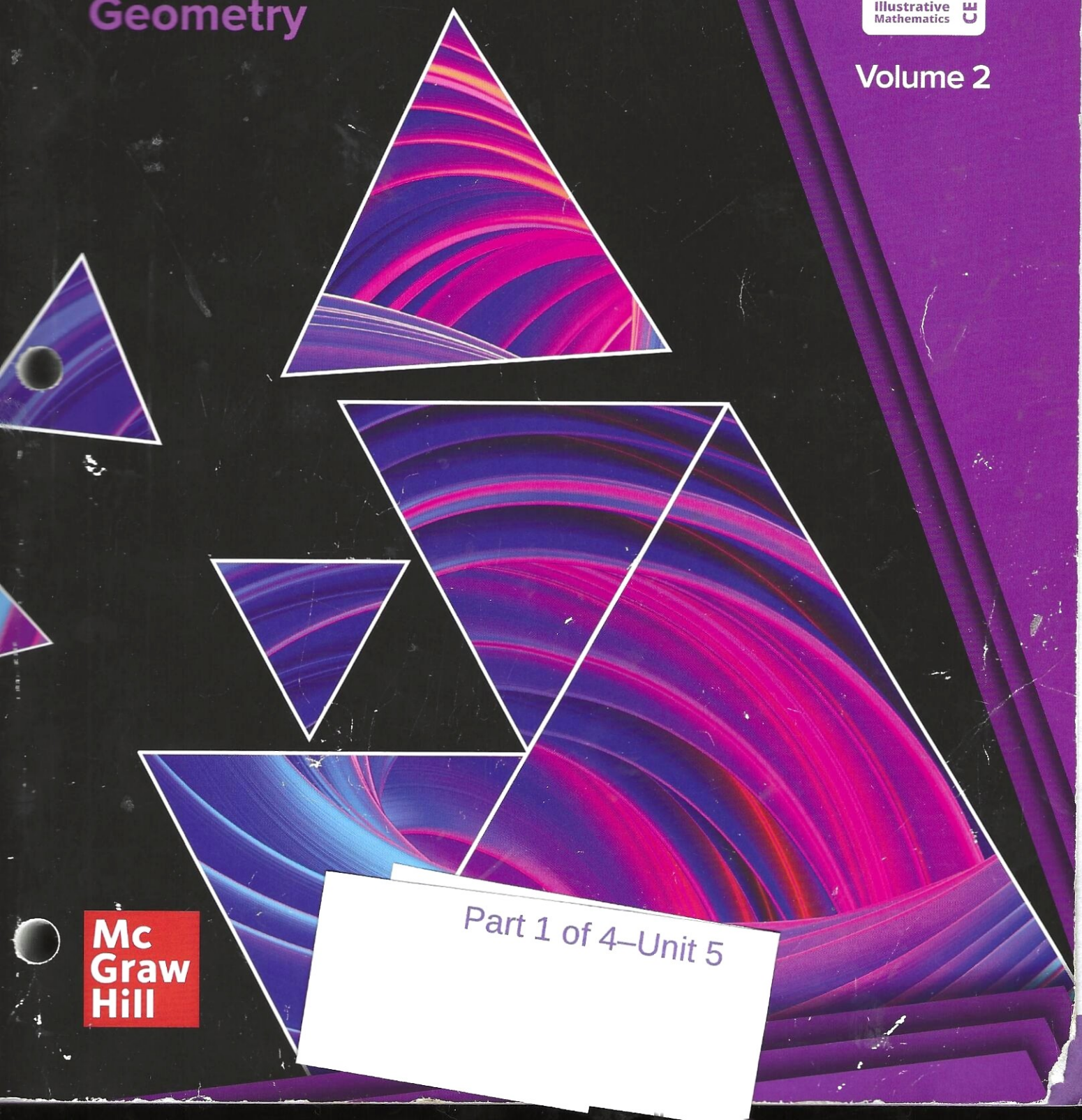
Illustrative Mathematics®

Geometry

Illustrative Mathematics CERTIFIED

Volume 2



Part 1 of 4–Unit 5

Mc Graw Hill

Volume 2

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Illustrative Mathematics®

Geometry

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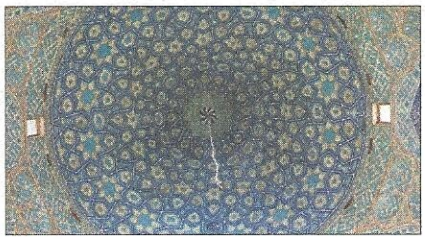
Contents in Brief



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Constructions and Rigid Transformations

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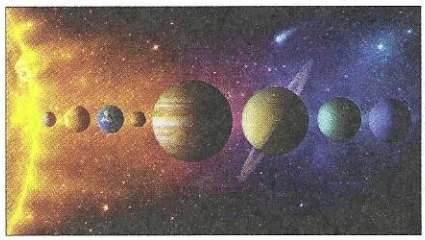
Congruence



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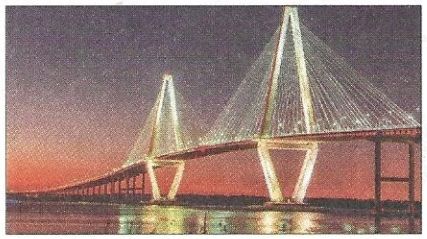
Similarity



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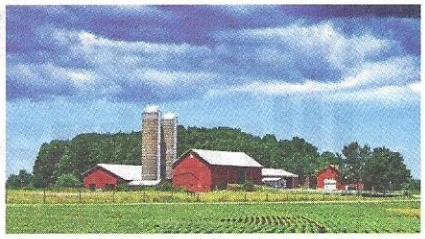

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Solid Geometry

Unit 6
Coordinate Geometry

Unit 7
Circles

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Illustrative Mathematics is designed to provide opportunities for you to learn math by doing math. By engaging in problem-solving activities, you will make and test your own conclusions and try out different strategies as you learn math.

Let's get started!

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Unit 5

Solid Geometry

MaxyM/Shutterstock

Knowing the volume of these cylindrical storage silos will help the farmer plan according to her needs. You'll learn more about geometric solids in this unit.

Topics

- Cross Sections, Scaling, and Area
- Scaling Solids
- Prism and Cylinder Volumes
- Understanding Pyramid Volumes
- Putting It All Together

Unit 5

Solid Geometry

Cross Sections, Scaling, and Area

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Scaling Solids

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Putting It All Together

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Lesson 5-1

Solids of Rotation

NAME Justis Benson DATE _____ PERIOD _____

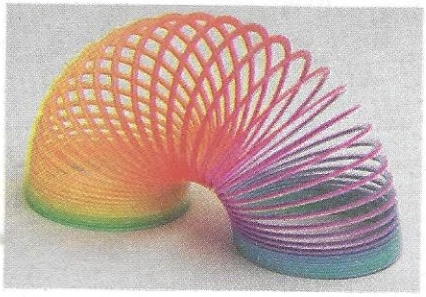
Learning Goal Let's rotate two-dimensional shapes to make three-dimensional shapes.

Warm Up

1.1 Which One Doesn't Belong: Solids

Which one doesn't belong?

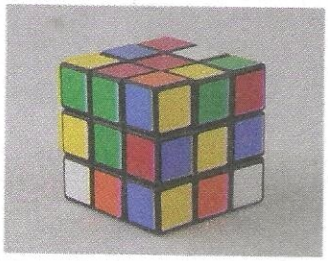
A



B

C

D



A because its hollow and it changes shapes

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Activity

1.2 Axis of Rotation

Your teacher will give you a shape. Tape 1 side of the shape to a pencil.

1. Spin the pencil between your hands. What solid is traced out as you rotate the shape? Draw the solid.

2. Predict what solids will be formed by the shapes of other members of your group. Confirm by asking them to rotate their shapes.

Are you ready for more?

1. Graph $y = -x + 3$ from $x = 1$ to $x = 3$.

2. Sketch the solid of rotation generated by rotating this line using the y-axis as the axis of rotation.

3. What figure is made?

4. The object being rotated here is a line, not a two-dimensional object like in the lesson. How does this affect the result of the rotation?

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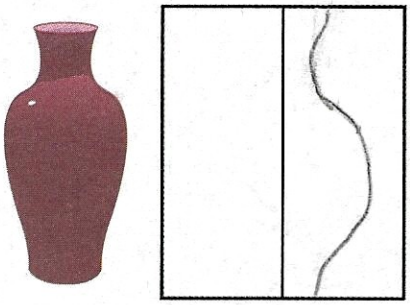
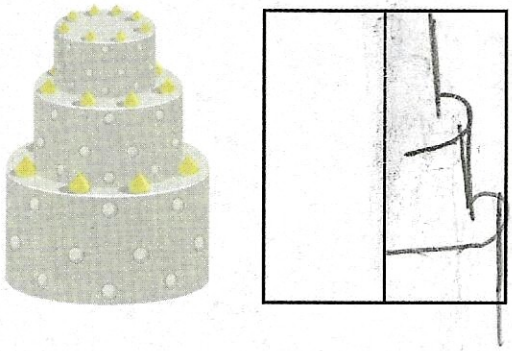
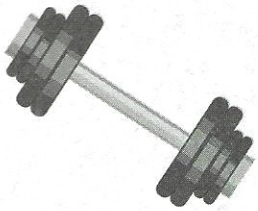
Activity

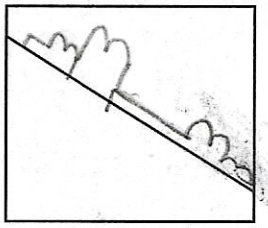
1.3 From Three Dimensions to Two

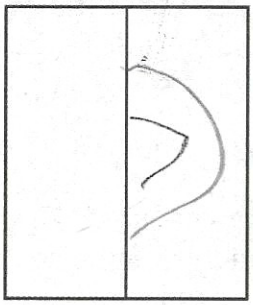
Draw the two-dimensional shape that, when rotated using the given **axis of rotation**, produces each **solid of rotation**. Ignore any non-symmetric aspects of the solid.







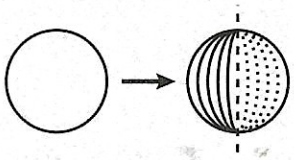




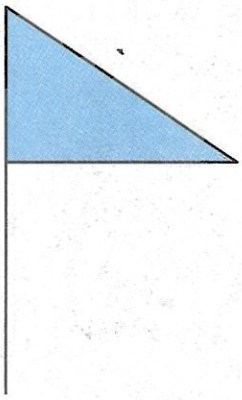
Summary

Solids of Rotation

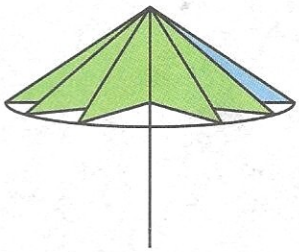
Take a coin and spin it on its edge. Give it good speed and make sure it stays vertical. What shape do you see? When you spin the coin, you should see a sphere.



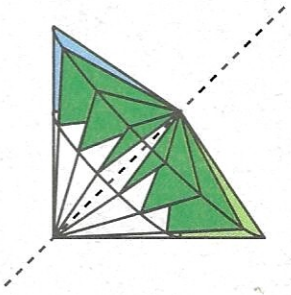
This triangular flag is made of metal.
What shape will you see if the pole is spun quickly?



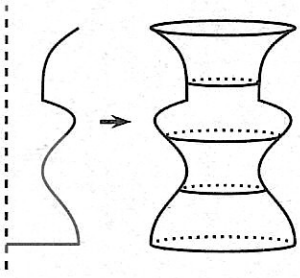
When the flag rotates with the pole, you should see a cone.



In both pictures, the object spins around a line called the **axis of rotation**, and the solid created is called the **solid of rotation**. If you were to spin the flag using a different axis of rotation, you would see a different solid of rotation. This is what it looks like when the flag is rotated using a different axis of rotation.



A machine called a lathe cuts away at a rotating block of material. For example, a lathe could be used to make a decorative sculpture shaped like the figure on the right from a block of wood. The image on the left shows the cross-sectional shape that would remain after the lathe carved away part of the spinning block of wood. If we rotate this two-dimensional shape using the vertical axis shown, it produces the sculpture shape.



Glossary

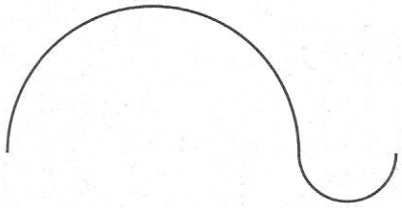
axis of rotation
solid of rotation

NAME _____ DATE _____ PERIOD _____

Practice

Solids of Rotation

1. Sketch the solid of rotation formed by rotating the given two-dimensional figure using the horizontal line shown as an axis of rotation.

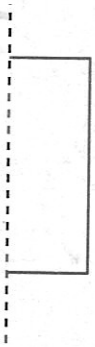


2. Draw a two-dimensional figure that could be rotated using a vertical axis of rotation to give the barrel shown.

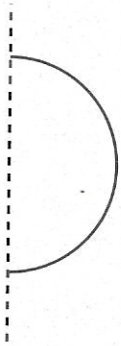
3. Match the two-dimensional figure and axis of rotation with the solid of rotation that can be formed by rotating the figure using that axis.

A.

B.



C.



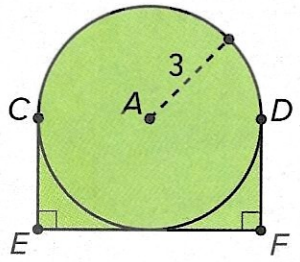
1. a cylinder

2. a sphere

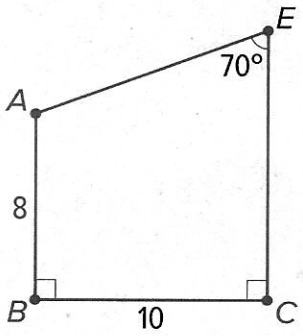
3. a cone

NAME _____ DATE _____ PERIOD _____

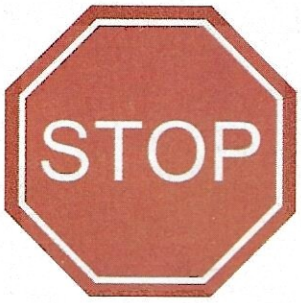
4. Find the area of the shaded region. **(Lesson 4-11)**



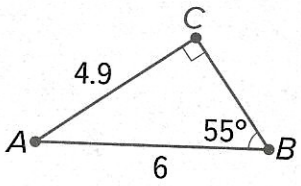
5. *Technology required.* Find the area of the figure: **(Lesson 4-11)**



6. *Technology required.* This stop sign is a regular octagon. It has side lengths of 12 inches. What is the area? What is the perimeter? **(Lesson 4-10)**



7. Right triangle ***ABC*** is shown. Select **all** expressions which are equal to the length of side ***BC***. **(Lesson 4-6)**



(A.) $\sqrt{4.9^2+6^2}$

(B.) $\sqrt{6^2-4.9^2}$

(C.) $4.9 \sin(55)$

(D.) $\frac{4.9}{\sin(55)}$

(E.) $4.9 \tan(55)$

(F.) $\frac{4.9}{\tan(55)}$

(G.) $6 \cos(55)$

(H.) $\frac{6}{\cos(55)}$

Lesson 5-2

Slicing Solids

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's analyze cross sections by slicing three-dimensional solids.

Warm Up

2.1 Slice This

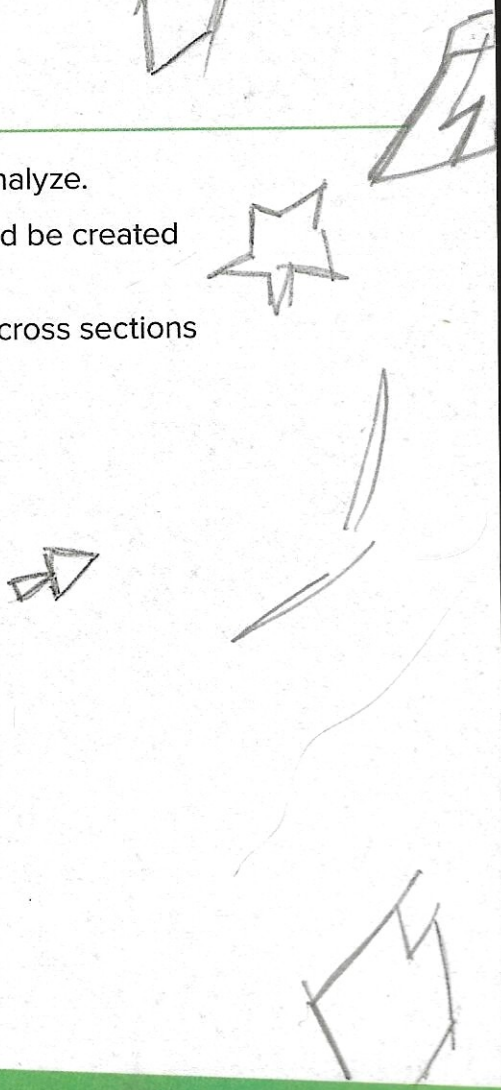
Imagine slicing a cylinder with a straight cut. The flat surface you sliced along is called a **cross section**. Try to sketch all the possible kinds of cross sections of a cylinder.

Activity

2.2 Slice That

Your teacher will give your group a three-dimensional solid to analyze.

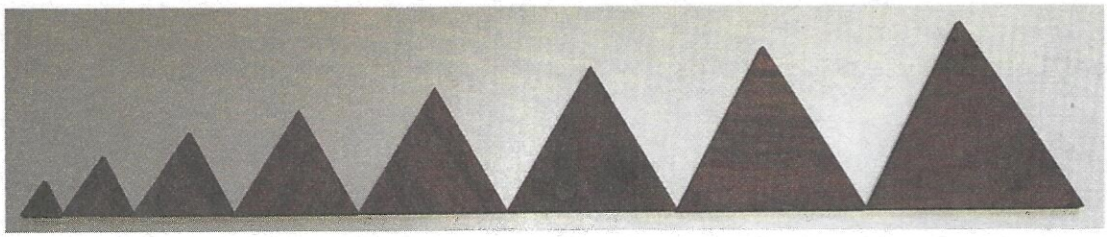
1. Sketch predictions of all the kinds of cross sections that could be created from your solid.
2. Slice your solid to confirm your predictions. Sketch any new cross sections you find after slicing.

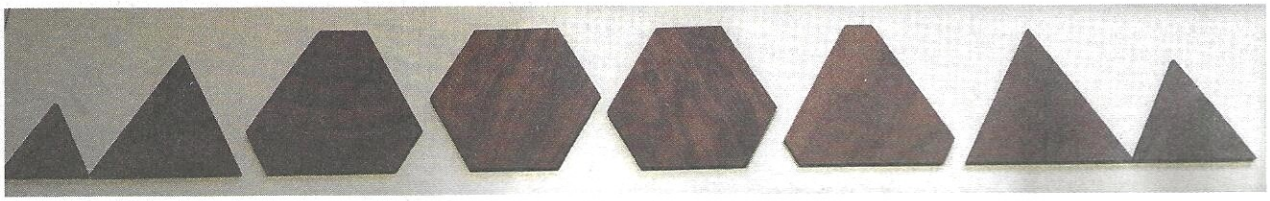


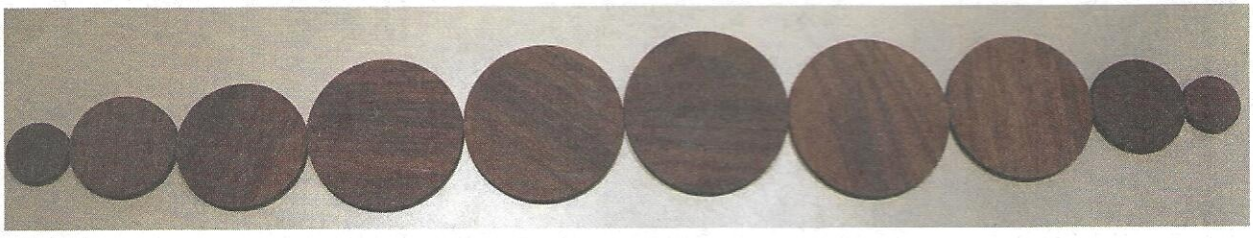
Activity

2.3 Stack 'Em Up

Each question shows several parallel cross-sectional slabs of the same three-dimensional solid. Name each solid.

1. 

2. 

3. 

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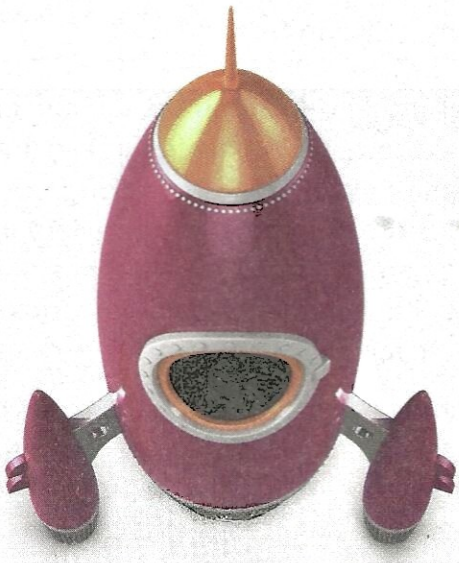
Are you ready for more?

3D-printers stack layers of material to make a three-dimensional shape. Computer software slices a digital model of an object into layers, and the printer stacks those layers one on top of another to replicate the digital model in the real world.



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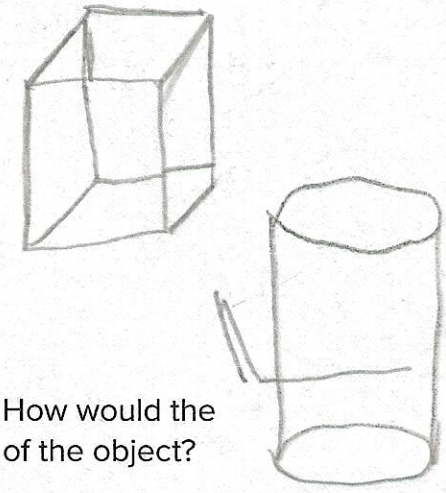
1. Draw 3 different horizontal cross sections from the object in the image.



2. The layers can be printed in different thicknesses. How would the thickness of the layers affect the final appearance of the object?

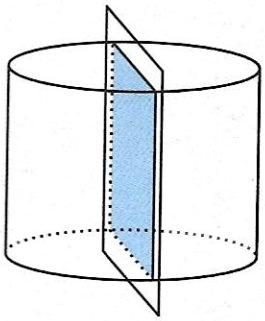
3. Suppose we printed a rectangular prism. How would the thickness of the layers affect the final appearance of the prism?

Summary

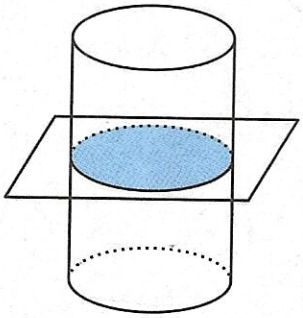
Slicing Solids

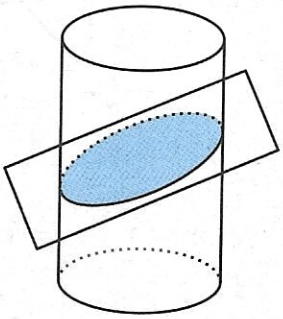
In earlier grades, you learned some vocabulary terms about solid geometry: A **sphere** is the set of points in three-dimensional space the same distance from some center. A **prism** has two congruent **faces** (or sides) that are called bases. The bases are connected by parallelograms. A **cylinder** is like a prism except the bases are circles. A **pyramid** has one base. The remaining faces are triangles that all meet at a single vertex. A **cone** is like a pyramid except the base is a circle.

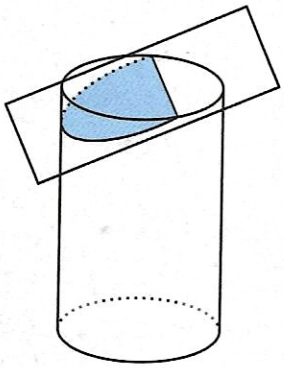
We often analyze **cross sections** of solids. A cross section is the intersection of a solid with a *plane*, or a two-dimensional figure that extends forever in all directions. For example, some cheese is sold in cylindrical blocks. If you stand the cheese on end and slice vertically, you will get a rectangle, as shown. This rectangle is a cross section of the cylinder.



Here are 3 more examples of cross sections created by intersecting a plane and a cylinder.

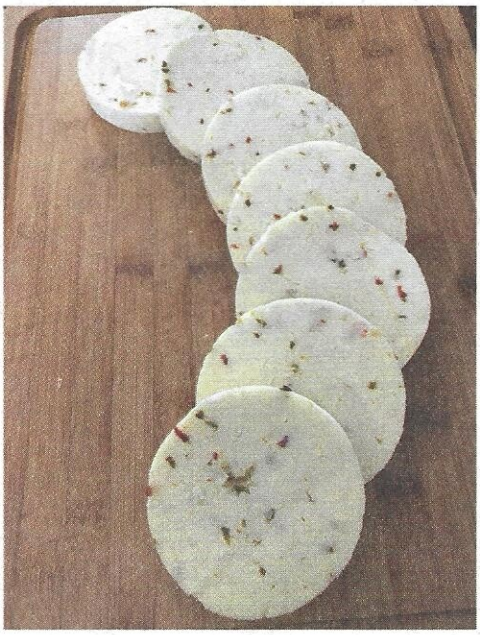






NAME _____ DATE _____ PERIOD _____

If you wanted to serve your cylindrical cheese at a party, you might cut it into several pieces, like this. The pieces are thin cylinders. They are like cross sections, but they are three-dimensional. All the cuts were made parallel to one another. By looking at the slices, or by stacking them up, you could figure out that the original shape of the cheese was a cylinder.



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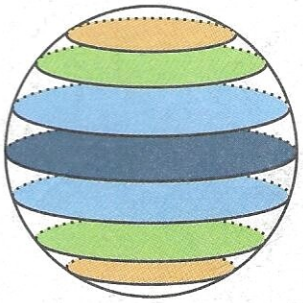
What if another cheese plate contained slices whose radii got bigger to a maximum size and then got smaller again? The cheese was probably in the shape of a sphere. A sphere has circular cross sections. The size of the circular cross sections increases as you get closer to the center of the sphere, then decreases past the center.



Glossary

cone
cross section
cylinder
face
prism
pyramid
sphere

Practice

Slicing Solids

1. Select **all** figures for which there exists a direction such that all cross sections taken at that direction are congruent.

   A. triangular pyramid

   B. square pyramid

   C. rectangular prism

   D. cube

   E. cone

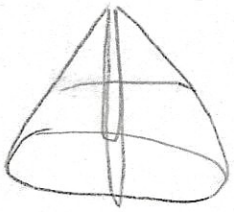
   F. cylinder

   G. sphere

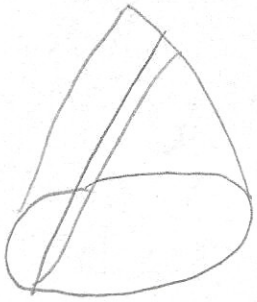
2. Imagine an upright cone with its base resting on your horizontal desk. Sketch the cross section formed by intersecting each plane with the cone.

   a. vertical plane not passing through the cone's topmost point

   b. horizontal plane



   c. diagonal plane



3. Name 2 figures for which a circle can be a cross section.

   Cone, Circle

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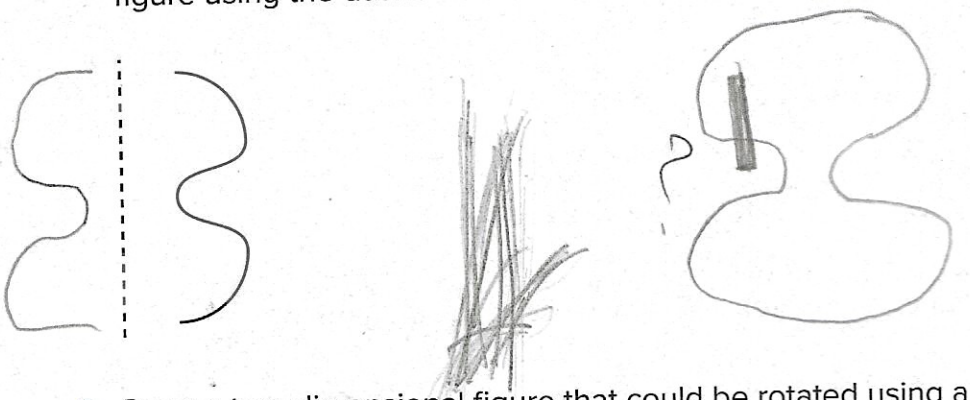
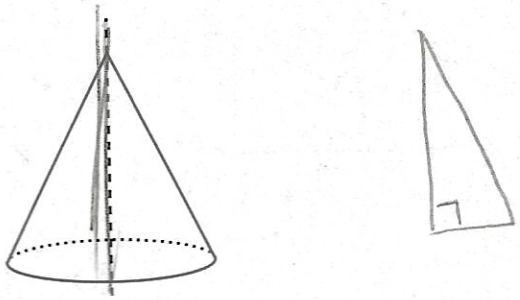
4. Sketch the solid of rotation formed by rotating the given two-dimensional figure using the dashed vertical line as an axis of rotation. (Lesson 5-1)



5. Draw a two-dimensional figure that could be rotated using a vertical axis of rotation to give the cone shown. (Lesson 5-1)



6. A regular hexagon and a regular octagon are both inscribed in the same circle. Which of these statements is true? (Lesson 4-11)

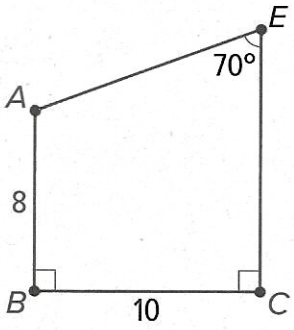
(A.) The perimeter of the hexagon is less than the perimeter of the octagon, and each perimeter is less than the circumference of the circle.

(B.) The perimeter of the octagon is less than the perimeter of the hexagon, and each perimeter is less than the circumference of the circle.

(C.) The perimeter of the hexagon is greater than the perimeter of the octagon, and each perimeter is greater than the circumference of the circle.

(D.) The perimeter of the octagon is greater than the perimeter of the hexagon, and each perimeter is greater than the circumference of the circle.

7. *Technology required.* Find the perimeter of the figure. **(Lesson 4-10)**



8. Match each trigonometric function to a ratio. You may use ratios more than once. **(Lesson 4-6)**

A. tan(*A*)

B. tan(*B*)

C. cos(*A*)

D. cos(*B*)

E. sin(*A*)

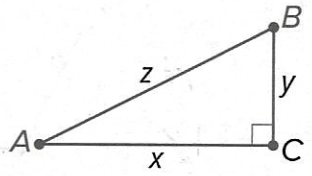
F. sin(*B*)

1. $\frac{y}{z}$

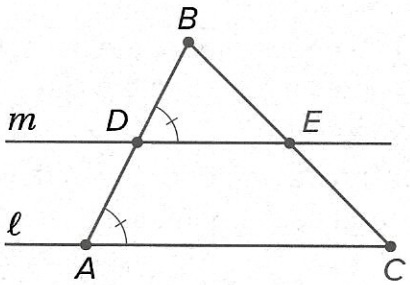
2. $\frac{x}{z}$

3. $\frac{x}{y}$

4. $\frac{y}{x}$



9. Explain how you know lines m and ℓ are parallel. **(Lesson 1-20)**



Lesson 5-3

Creating Cross Sections by Dilating

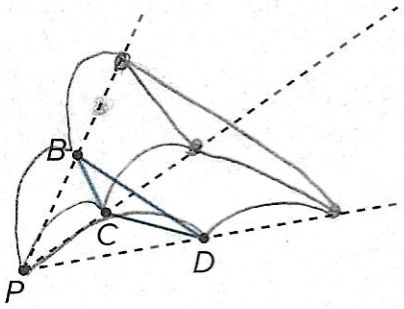
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's create cross sections by doing dilations.

Warm Up

3.1 Dilating, Again

Dilate triangle *BCD* using center *P* and a scale factor of 2.



Look at your drawing. What do you notice? What do you wonder?

I dont care

Activity

3.2 Pyramid Mobile

Your teacher will give you sheets of paper. Each student in the group should take one sheet of paper and complete these steps:

1. Locate and mark the center of your sheet of paper by drawing diagonals or another method.
2. Each student should choose one scale factor from the table. On your paper, draw a dilation of the entire sheet of paper, using the center you marked as the center of dilation.
3. Measure the length and width of your dilated rectangle and calculate its area. Record the data in the table.
4. Cut out your dilated rectangle and make a small hole in the center.

Scale Factor, k	Length of Scaled Rectangle	Width of Scaled Rectangle	Area of Scaled Rectangle
$k = 0.25$	2.125	2.75	5.84
$k = 0.5$	4.25	5.5	9.75
$k = 0.75$	6.375	8.25	14.625
$k = 1$	8.5	11	935

Illustrative Math

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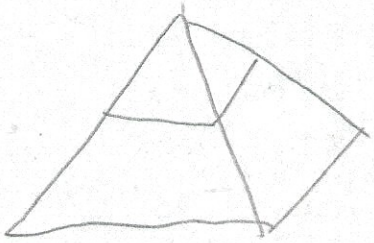
Now the group as a whole should complete the remaining steps:

1. Cut 1 long piece of string (more than 30 centimeters) and 4 shorter pieces of string. Make 4 marks on the long piece of string an equal distance apart.
2. Thread the long piece of string through the hole in the largest rectangle. Tie a shorter piece of string beneath it where you made the first mark on the string. This will hold up the rectangle.
3. Thread the remaining pieces of paper onto the string from largest to smallest, tying a short piece of string beneath each one at the marks you made.
4. Hold up the end of the string to make your cross sections resemble a pyramid. As a group, you may have to steady the cross sections for the pyramid to clearly appear.

Are you ready for more?

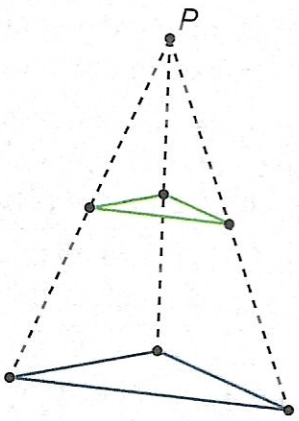
Is dilating a square using a factor of 0.9, then dilating the image using scale factor 0.9 the same as dilating the original square using a factor of 0.8? Explain or show your reasoning.



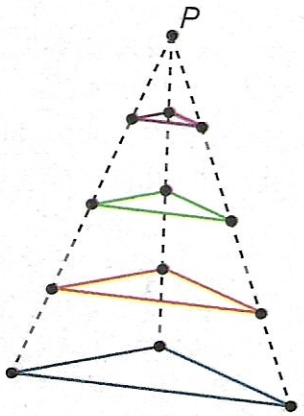
Summary

Creating Cross Sections by Dilating

Imagine a triangle lying flat on your desk, and a point P directly above the triangle. If we dilate the triangle using center P and scale factor $k = \frac{1}{2}$ or 0.5, together the triangles resemble cross sections of a pyramid.



We can add in more cross sections. This image includes two more cross sections, one with scale factor $k = 0.25$ and one with scale factor $k = 0.75$. The triangle with scale factor $k = 1$ is the base of the pyramid, and if we dilate with scale factor $k = 0$ we get a single point at the very top of the pyramid.



Each triangle's side lengths are a factor of k times the corresponding side length in the base. For example, for the cross section with $k = \frac{1}{2}$, each side length is half the length of the base's side lengths.

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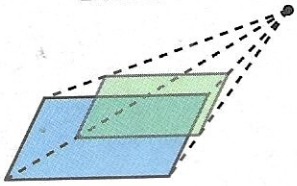
Practice

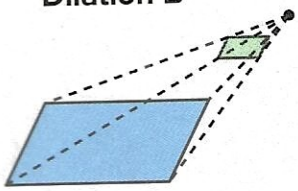
Creating Cross Sections by Dilating

1. Each image shows a quadrilateral in a plane. The quadrilateral has been dilated using a center above the plane and a scale factor between 0 and 1. Match the dilation with the scale factor used.

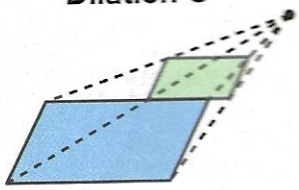
Dilation A



Dilation B



Dilation C



A. Dilation A

B. Dilation B

C. Dilation C

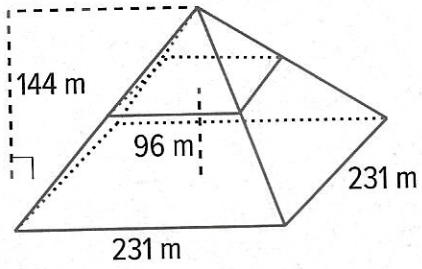
1. $\frac{1}{4}$

2. $\frac{1}{2}$

3. $\frac{3}{4}$

2. The Pyramid of Khufu in Giza, Egypt was the world's tallest free-standing structure for more than 3,500 years. Its original height was about 144 meters. Its base is approximately a square with a side length of 231 meters.

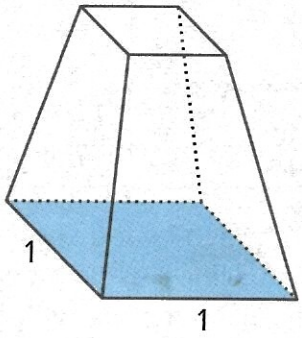
The diagram shows a cross section created by dilating the base using the top of the pyramid as the center of dilation. The cross section is at a height of 96 meters.



a. What scale factor was used to create the cross section?

b. What are the dimensions of the cross section?

3. The horizontal cross sections of this figure are dilations of the bottom rectangle using a point above the rectangle as a center. What scale factors of dilation are represented in the figure's cross sections?



(A.) scale factors between 0 and $\frac{1}{2}$

(B.) scale factors between 0 and 1

(C.) scale factors between $\frac{1}{4}$ and $\frac{3}{4}$

(D.) scale factors between $\frac{1}{2}$ and 1

4. Imagine an upright cone with its base resting on your horizontal desk. Match each plane with the image of the cross section formed by intersecting the plane with the cone. (Lesson 5-2)

Figure 1

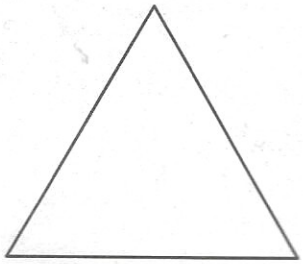


Figure 2

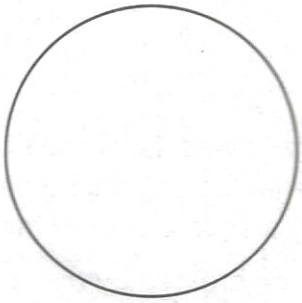
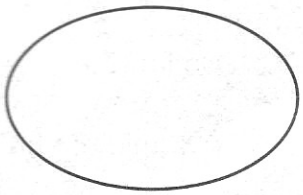


Figure 3



A. horizontal

B. vertical, through cone's topmost point

C. diagonal

1. Figure 1

2. Figure 2

3. Figure 3

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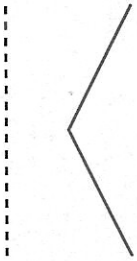
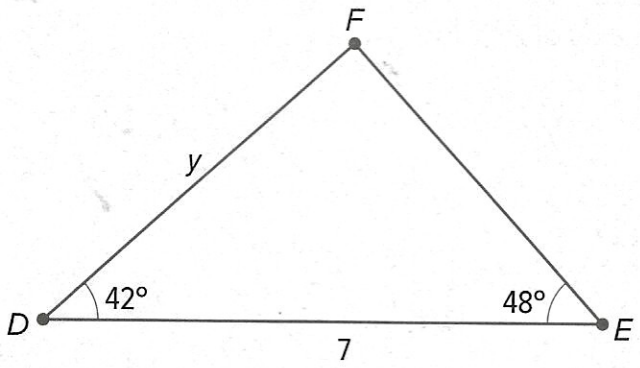
5. What is the shape of the cross section formed by intersecting a cube with a vertical plane that passes through opposite edges of the cube? Explain how you know. (Lesson 5-2)

6. Sketch the solid of rotation formed by rotating the given two-dimensional figure using the dashed vertical line as an axis of rotation. (Lesson 5-1)



7. *Technology required.* A rope with a length of 4 meters is tied from a stake in the ground to the top of a tent. It forms a 20 degree angle with the ground. How tall is the tent? **(Lesson 4-7)**

8. *Technology required.* What is the value of y? **(Lesson 4-6)**



Lesson 5-4

Scaling and Area

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's see how the area of shapes changes when we dilate them.

Warm Up

4.1 Squares and Roots

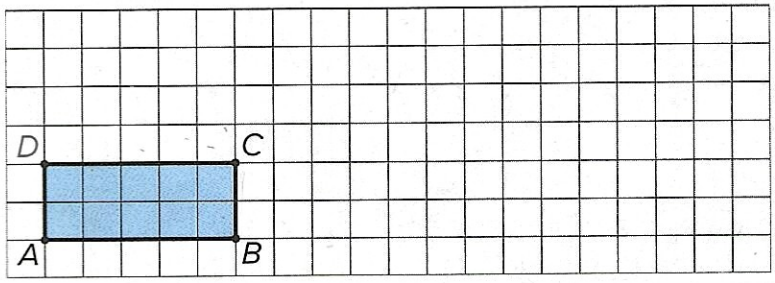
1. What number times itself equals 25? 5
2. What number times itself equals 81? 9
3. What number times itself equals 10? 3.2

Activity

4.2 Scaling Up a Rectangle

Here is a rectangle with length 5 units and width 2 units.

A = L · W



1. What is the area of the rectangle? 5 x 2 = 10
2. Dilate rectangle *ABCD* from point *A* by a scale factor of 2. Calculate the area of the image. L = 10 5x2 W = 4 2x2 40
3. Dilate rectangle *ABCD* from point *A* by a scale factor of 3. Calculate the area of the image. 5x3=15 2x3=6 15x6=90

4. Complete the table.

Scale Factor	Area of Image in Square Units	Factor by Which the Area Changed
0.5	2.5	
1	10	
2	40	
2.5	62.5	
3	90	
4	160	

$\frac{\text{New Area}}{\text{Original Area}}$

5. Write an expression for the area of a rectangle with length ℓ and width w.

$A = \ell \cdot w$

6. Imagine dilating the rectangle with length ℓ and width w by a factor of k. Write expressions for the dimensions of the dilated rectangle.

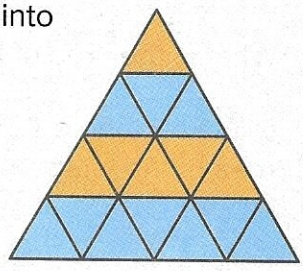
7. Write an expression for the area of the dilated rectangle.

8. Use your work to draw a conclusion about what happens to the area of a rectangle when it's dilated by a scale factor of k.

Are you ready for more?

The image shows a dilation of an equilateral triangle by scale factors of 1, 2, 3, and 4. At each stage, the dilated shape is partitioned into triangles congruent to the original.

1. Why is the difference in the number of original triangles that fit into the image between 2 successive scale factors always odd?



2. What does this image tell us about the sum of the first n odd numbers?

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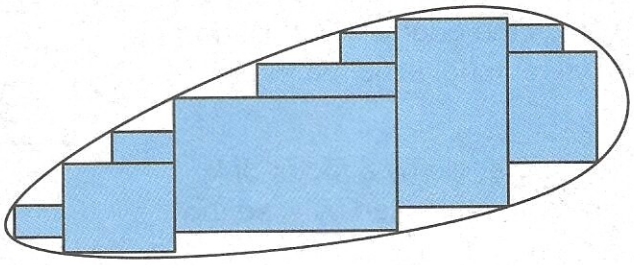
Activity

4.3 What About Other Shapes?

Andre says, "We know that if a rectangle is scaled by a factor of k, the area scales by a factor of k^2. Does this apply to other shapes?"

Jada says, "Here's a shape that's not a rectangle. Say its area is A square units. Let's draw some rectangles on it that get smaller and smaller to fit the remaining empty space. With enough rectangles we can come close to covering the whole blob."



Andre says, "These rectangles start to make a nice approximation of the blob. If we wanted to get closer, we could add even more rectangles. The sum of the areas of all the rectangles would add up to the area of the blob. I think we're almost there!"

1. Suppose the blob is dilated by a factor of k. In doing this, the rectangles covering the blob also get dilated by a factor of k. How does the area of each dilated rectangle compare to the area of each original rectangle?

Multiply Original area by k^2

2. What does this tell you about the area of the dilated image? Explain your reasoning.

The Area of the blob is also times k^2

3. Suppose a circle has area 20 square inches and it's dilated using a scale factor of 6. What is the area of the image? Explain or show your reasoning.

$A' = k^2 \cdot A$

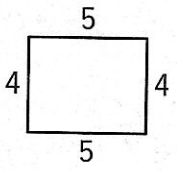
$A' = 6^2 \cdot 20$

$= 36 \cdot 20 = 720 \text{ in}^2$

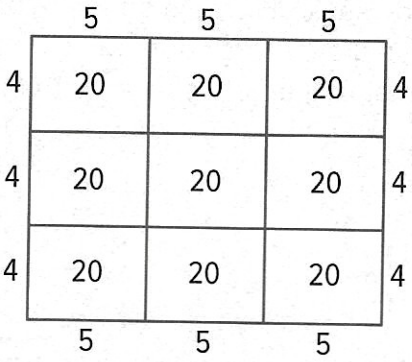
Summary

Scaling and Area

Say you have a 5 inch by 4 inch rectangle. Its area is 20 square inches. If you dilate the rectangle by a scale factor of 3, what do you think the area of the new rectangle will be?

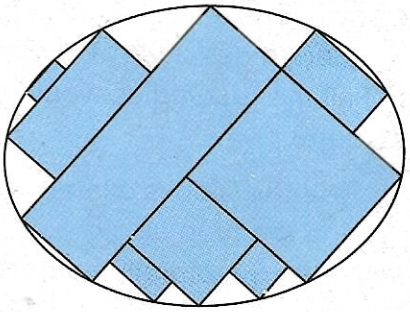


The dilated rectangle's dimensions can be written as $3 \cdot 5$ and $3 \cdot 4$. Substitute these values into the area expression $\ell \cdot w$ to get $(3 \cdot 5)(3 \cdot 4)$. Rearrange the numbers to get $(3 \cdot 3)(5 \cdot 4)$. This can be rewritten $(3^2)(5 \cdot 4)$ or $9 \cdot 20$. So the area of the dilated rectangle is 180 square inches, or 9 times the original. The diagram confirms that 9 of the original rectangles fit into the dilated rectangle.



In general, when you scale a rectangle by a factor of k, the length and the width are both multiplied by k, so the area is multiplied by k^2.

What if you dilate a shape that is not a rectangle by a scale factor of k? Consider the rounded shape called an *ellipse* in the image. You can approximate the area of an ellipse by filling it with many rectangles. The sum of the areas of the rectangles will be a little less than the area of the ellipse because they don't fill it entirely. If you want to get closer to the area of the ellipse, draw in more rectangles. If you continued the process infinitely, you could find the exact area of the ellipse this way.



If you dilated this ellipse using a scale factor of 4, each rectangle would become 16 times larger. This means that the area of the ellipse will increase by a factor of 16 as well. Any closed shape can be filled with rectangles that approximate its area. Because of this, when you scale *any* shape by a factor of k, its area is multiplied by a factor of k^2.

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Practice

Scaling and Area

1. A rectangle with area 12 square units is dilated by a scale factor of k. Find the area of the image for each given value of k.

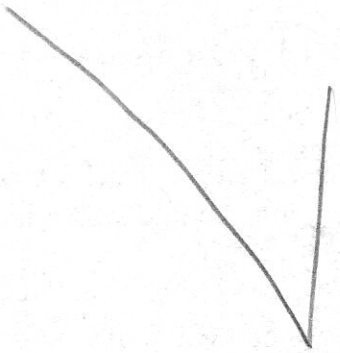
   a. $k = 2$ $12 \rightarrow 24$

   b. $k = 5$ $12 \rightarrow 60$

   c. $k = 1$ $12 \rightarrow 12$

   d. $k = \frac{1}{4}$ $12 \rightarrow 3$

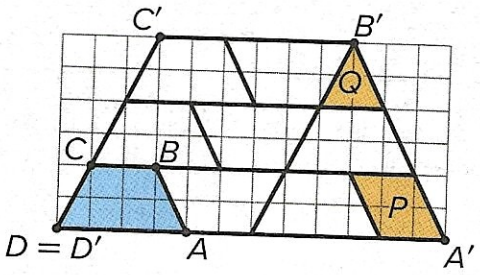
   e. $k = 1.2$ $12 \rightarrow 14.4$



2. The area of a circle of radius 1 is π units squared. Use scaling to explain why the area of a circle of radius r is πr^2 units squared.

? The Area of a circle is the same as the radius

3. Trapezoid $A'B'C'D$ was created by dilating trapezoid $ABCD$ using D as the center of dilation.



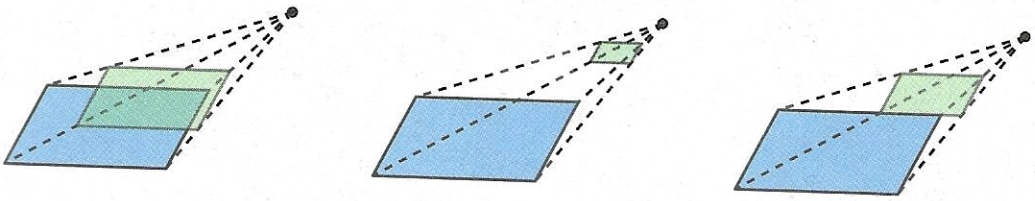
a. What was the scale factor of the dilation? 3

b. Based on the scale factor, how many copies of $ABCD$, including the original, should fit inside $A'B'C'D'$? 9

c. How can you see your answer to these questions in the diagram? Look at the image above

4. Each image shows a quadrilateral in a plane. The quadrilateral has been dilated using a center above the plane and a scale factor between 0 and 1. Estimate the scale factor that was used for each dilation. (Lesson 5-3)

Dilation A **Dilation B** **Dilation C**



The scale factor is 2

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5. Select the solid whose cross sections are dilations of some two-dimensional shape using a point directly above the shape as a center and scale factors ranging from 0 to 1. (Lesson 5-3)

(A.) cone

(B.) cube

(C.) cylinder

(D.) triangular prism

6. Select **all** figures for which at least one cross section is a circle. (Lesson 5-2)

(A.) triangular pyramid

(B.) square pyramid

(C.) rectangular prism

(D.) cube

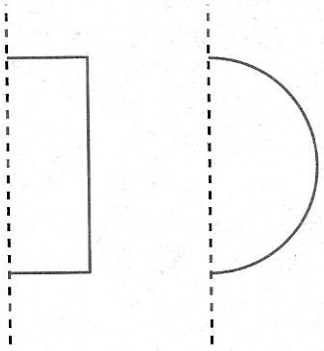
(E.) cone

(F.) cylinder

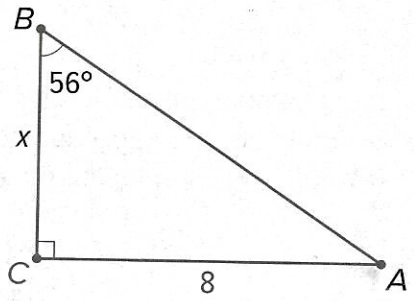
(G.) sphere

7. If the two-dimensional figures are rotated around the vertical axes of rotation shown, what solids are formed? (Lesson 5-1)

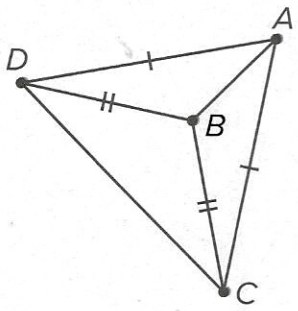
Square and A Circle



8. Tyler and Jada wish to find the value of x, the length of side BC in this triangle. Tyler decides to set up the equation $\tan(56) = \frac{8}{x}$. Jada says she prefers an equation that has x in the numerator. What is an equation she could use instead? **(Lesson 4-8)**



9. Triangles ACD and BCD are isosceles. Angle DBC has a measure of 110 degrees and angle BDA has a measure of 22 degrees. Find the measure of angle BAC.
$\overline{AD} \cong \overline{AC}$ and $\overline{BD} \cong \overline{BC}$ **(Lesson 2-6)**



Lesson 5-5

Scaling and Unscaling

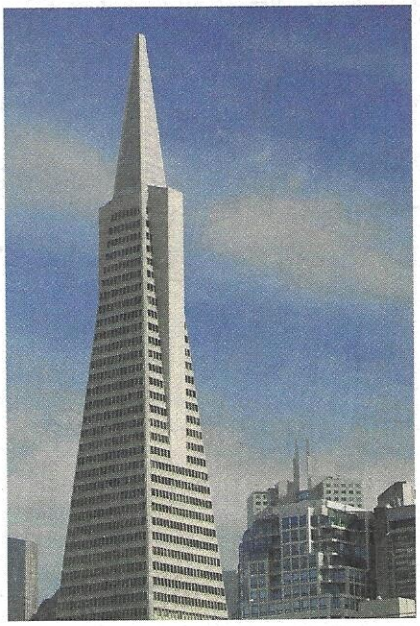
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's examine the relationships between areas of dilated figures and scale factors.

Warm Up

5.1 Transamerica Building

The image shows the Transamerica Building in San Francisco. It's shaped like a pyramid.



George Doyle/Stockbyte/Getty Images

The bottom floor of the building is a rectangle measuring approximately 53 meters by 44 meters. The top floor of the building is a dilation of the base by scale factor $k = 0.32$.

Ignoring the triangular "wings" on the sides, what is the area of the top floor? Explain or show your reasoning.

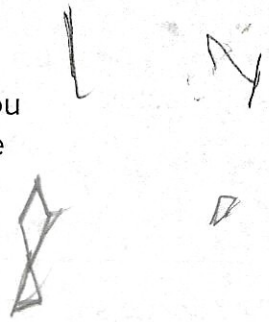
Activity

5.2 Two Viewpoints

A triangle has area 100 square inches. It's dilated by a factor of $k = 0.25$.

Mai says, "The dilated triangle's area is 25 square inches."

Lin says, "The dilated triangle's area is 6.25 square inches."



1. For each student, decide whether you agree with their statement. If you agree, explain why. If you disagree, explain what the student may have done to arrive at their answer.

2. Calculate the area of the image if the original triangle is dilated by each of these scale factors:

   a. $k = 9$ $9^2 \cdot 100$

   $81 \cdot 100 = 8100 \text{ in}^2$

   b. $k = \frac{3}{4}$ $\left(\frac{3}{4}\right)^2 \cdot 100 = 56.25 \text{ in}^2$

   .5625

Activity

5.3 Graphing Areas and Scale Factors

An artist painted a 1 foot square painting. Now she wants to create more paintings of different sizes that are all scaled copies of her original painting. The paint she uses is expensive, so she wants to know the sizes she can create using different amounts of paint.

3=k

1. Suppose the artist has enough paint to cover 9 square feet. If she uses all her paint, by what scale factor can she dilate her original painting?

2. Complete the table that shows the relationship between the dilated area (x) and the scale factor (y). Round values to the nearest tenth if needed.

Dilated Area in Square Feet	Scale Factor
0	0
1	1
4	2
9	3
16	4

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3. On graph paper, plot the points from the table and connect them with a smooth curve.

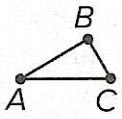
4. Use your graph to estimate the scale factor the artist could use if she had enough paint to cover 12 square feet.

5. Suppose the painter has enough paint to cover 1 square foot, and she buys enough paint to cover an additional 2 square feet. How does this change the scale factor she can use?

6. Suppose the painter has enough paint to cover 14 square feet, and she buys enough paint to cover an additional 2 square feet. How does this change the scale factor she can use?

Are you ready for more?

The image shows triangle *ABC*.



1. Sketch the result of dilating triangle *ABC* using a scale factor of 2 and a center of *A*. Label it *AB′C′*. 3.5

2. Sketch the result of dilating triangle *ABC* using a scale factor of -2 and a center of *A*. Label it *AB″C″*.

3. Find a transformation that would take triangle *AB′C′* to *AB″C″*.

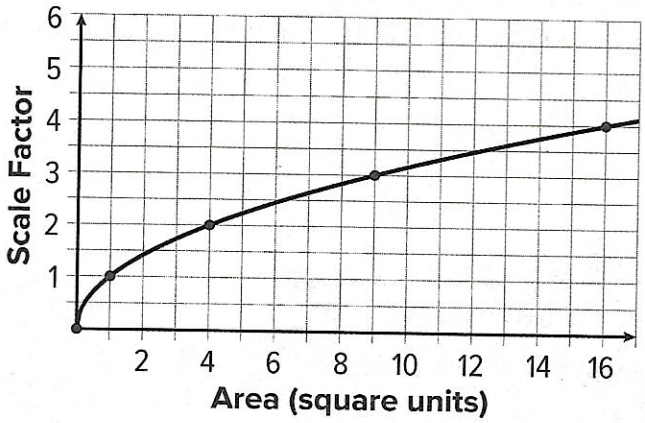
Summary

Scaling and Unscaling

If we know the area of an original figure and its dilation, we can work backwards to find the scale factor. For example, suppose we have a circle with area 1 square unit, and a dilation of the circle with area 64 square units. We know the circle must have been dilated by a factor of 8, because $8^2 = 64$. Another way to say this is $\sqrt{64} = 8$.

A graph can help us understand the relationship between dilated areas and scale factors. We can make a table of values for the dilated circle, plot the points on a graph, and connect them with a smooth curve. In this table, the dilated area is the input or x value, and the scale factor is the output or y value. Remember that the area of the original circle is 1 square unit, so the square root of the dilated area is the same as the scale factor.

Dilated Area in Square Units	Scale Factor
0	0
1	1
4	2
9	3
16	4



This graph represents the equation that describes the relationship between area and scale factor: $y = \sqrt{x}$. Note that the rate of change isn't constant. On the left side, the graph is fairly steep. As the area increases, the scale factor increases quickly. But on the right side, the graph flattens out. As the area continues to increase, the scale factor still increases, but not as quickly.

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Practice

Scaling and Unscaling

1. A circle with an area of 8π square centimeters is dilated so that its image has an area of 32π square centimeters. What is the scale factor of the dilation?

   (A.) 2

   (B.) 4

   (C.) 8

   (D.) 16

2. A trapezoid has an area of 100 square units. What scale factor would be required to dilate the trapezoid to have each area?

   a. 6400 square units

   64

   b. 900 square units

   9

   c. 100 square units

   1

   d. 25 square units

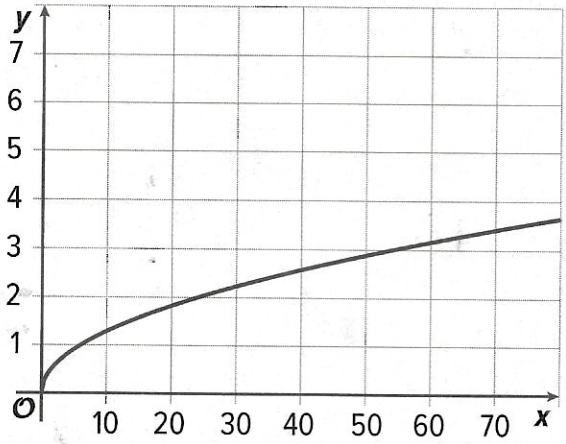
   2.5

   e. 4 square units

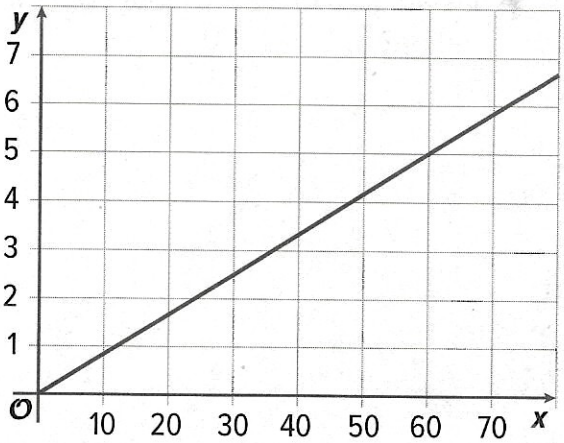
   0.04

3. A triangle has an area of 6 square inches and a perimeter of 12 inches. Suppose it is dilated by some scale factor, and the area and perimeter of the image are calculated. Match each graph with the relationship it represents.

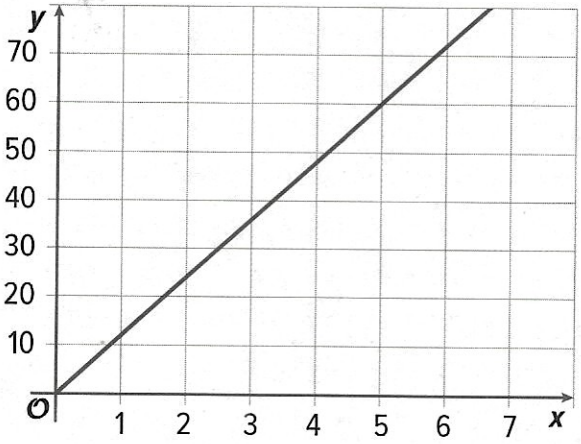
Graph A



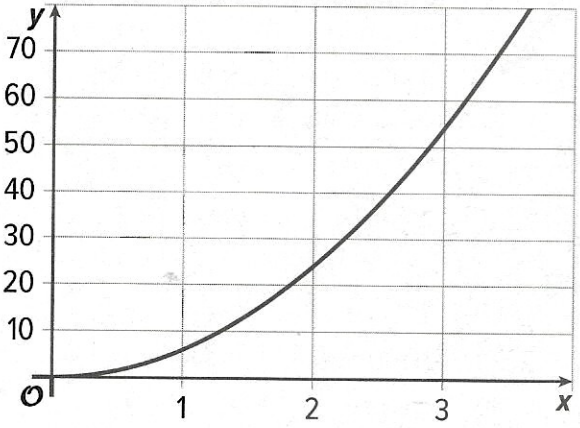
Graph B



Graph C



Graph D



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. scale factor is the *x*-value; perimeter is the *y*-value

2. scale factor is the *x*-value; area is the *y*-value

3. perimeter is the *x*-value; scale factor is the *y*-value

4. area is the *x*-value; scale factor is the *y*-value

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4. A polygon with area 10 square units is dilated by a scale factor of k. Find the area of the image for each value of k. **(Lesson 5-4)**

a. $k = 4$

40

b. $k = 1.5$

15

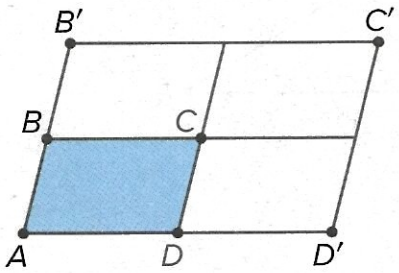
c. $k = 1$

10

d. $k = \frac{1}{3}$

3.33

5. Parallelogram $AB'C'D'$ was obtained by dilating parallelogram $ABCD$ using A as the center of dilation. **(Lesson 5-4)**



a. What was the scale factor of the dilation?

2

b. How many congruent copies of $ABCD$ have we fit inside $AB'C'D'$?

4

c. How does the area of parallelogram $AB'C'D'$ compare to parallelogram $ABCD$?

doubles in size

d. If parallelogram $ABCD$ has area 12 square units, what is the area of parallelogram $AB'C'D'$?

24

6. Select **all** solids whose cross sections are dilations of some two-dimensional shape using a point directly above the shape as a center and scale factors ranging from 0 to 1. (Lesson 5-3)

(A.) cylinder

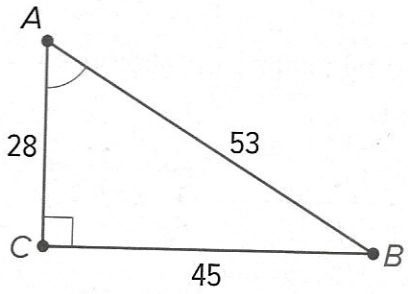
(B.) cube

(C.) triangular prism

(D.) cone

(E.) triangular pyramid

7. Select **all** expressions which give the measure of angle A. (Lesson 4-9)



(A.) $\arccos\left(\frac{28}{53}\right)$

(B.) $\arccos\left(\frac{45}{53}\right)$

(C.) $\arcsin\left(\frac{28}{53}\right)$

(D.) $\arcsin\left(\frac{45}{53}\right)$

(E.) $\arctan\left(\frac{28}{45}\right)$

(F.) $\arctan\left(\frac{45}{28}\right)$

Lesson 5-6

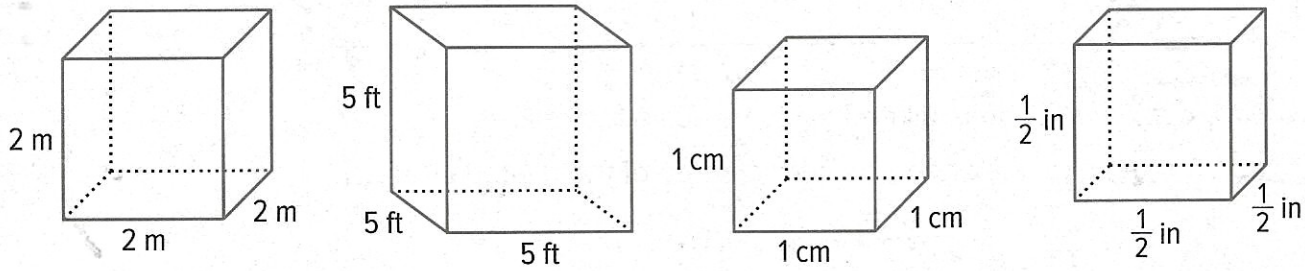
Scaling Solids

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's see how the surface area and volume of solids change when we dilate them.

Warm Up

6.1 Math Talk: Cube Volumes



Find the volume of each cube mentally.

Activity

6.2 How Do Surface Area and Volume Change with Scaling?

1. Use unit cubes to build cubes that result from dilating a unit cube by each scale factor shown in the table. Then, complete the table with the surface area and volume of each dilated cube.

Scale Factor	Surface Area in Square Units	Volume in Cubic Units
1	$6(1\times1)=6$	$1\cdot1\cdot1=1$
2	$6(2\times2)=24$	$2\cdot2\cdot2=8$
3	$6(3\times3)=54$	$3\cdot3\cdot3=27$
4	$6(4\times4)=96$	$4\cdot4\cdot4=64$

2. Suppose a unit cube is dilated by some scale factor k.

   a. Write an expression for the surface area of the dilated cube.

   $SA=6\cdot k^2$

   b. Write an expression for the volume of the dilated cube.

   $V=k^3$

   c. Compare and contrast the expression for surface area and the expression for volume.

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Activity

6.3 Scaling All Solids

Clare says, "We know that if we dilate a cube by a factor of k, the cube's volume is multiplied by k^3. It seems like that must apply to *all* solids, but I'm not sure how to prove it."

Elena says, "Earlier in the unit, we showed that we can cover any two-dimensional shape with rectangles, so the property that area changes by k^2 when we dilate a figure by k applies to all shapes, not just rectangles. Can we do something similar here?"

1. Use Elena's line of reasoning to argue that for *any* solid, if it's dilated by a factor of k, the volume is multiplied by k^3.

we can fill a shape with dilated cubes so this tells us any solid can by multiplied by $s*3$ find new volume

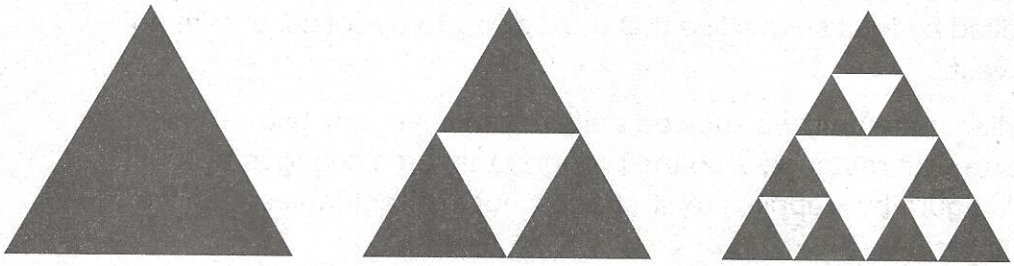
2. Suppose a triangular prism has surface area 84 square centimeters and volume 36 cubic centimeters. The prism is dilated by scale factor $k = 4$. Calculate the surface area and volume of the dilated prism.

$V' = V \cdot k^3 = 36 \cdot 4^3 = 36 \cdot 64$
$= 2304 cm^3$

Are you ready for more?

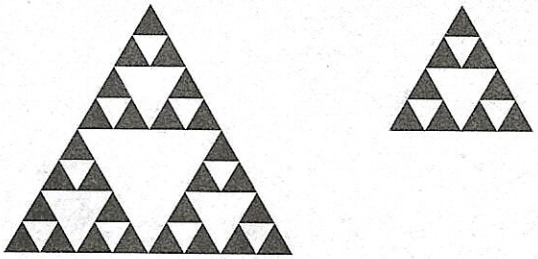
The image shows a figure called Sierpinski's triangle. It's formed by starting with an equilateral triangle, then repeatedly removing equilateral triangles created by joining the midpoints of the existing triangle's sides. The first few stages are shown.

stage 1 **stage 2** **stage 3**



If we continue this process of removal forever, we are left with some points that never get removed from the triangle. The remaining points are what we call Sierpinski's triangle. At any given stage, the triangle at the top of the figure is a scaled copy of the triangle at the previous stage.

stage 4 **top triangle**



For the completed Sierpinski's triangle, though, the top triangle is a scaled copy not of the previous stage, but of the full Sierpinski's triangle.

1. For the completed figure, what scale factor takes Sierpinski's triangle to its scaled copy at the top?
2. Based on the scale factor, what fraction of the original shaded region should be contained in the scaled copy at the top?
3. The scaled copy at the top actually contains $\frac{1}{3}$ of the shading of the original. Provide reasoning that shows that this is true.

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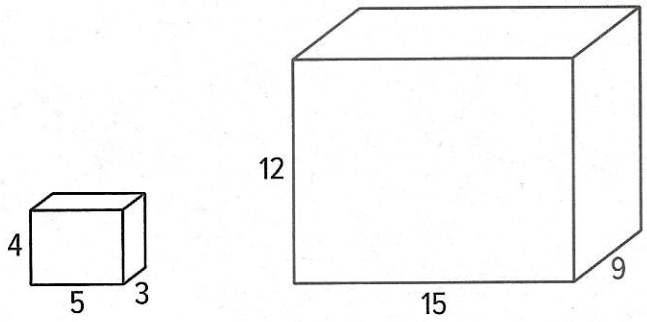
Summary

Scaling Solids

In earlier activities, we saw that if we dilate a two-dimensional shape, the area of the dilated shape is the area of the original shape multiplied by the square of the scale factor. What happens when we dilate three-dimensional solids?

Here is a rectangular prism with side lengths 3, 4, and 5 units. When we dilate the prism using a scale factor of 3, the lengths become 9, 12, and 15 units.



Since these are three-dimensional shapes, we can look at both volume and surface area. The volume of the original prism is 60 cubic units because $3 \cdot 4 \cdot 5 = 60$. The volume of the dilated prism is 1,620 cubic units because $9 \cdot 12 \cdot 15 = 1{,}620$.The volume became 27 times larger! Why? Since the side lengths tripled, when we calculated the volume we were really finding $(3 \cdot 3) \cdot (4 \cdot 3) \cdot (5 \cdot 3) = (3 \cdot 4 \cdot 5) \cdot 3^3$. The volume was multiplied by the *cube* of the scale factor, or by $3^3 = 27$.

Now let's look at surface area. How do you think the surface area will change when the prism is dilated?

In the original prism, 2 faces have area 12 square units, 2 have area 20 square units, and 2 have area 15 square units for a total surface area of 94 square units. The corresponding faces of the dilated prism have areas 108, 180, and 135 square units. The surface area of the new prism totals 846 square units, 9 times that of the original. Just like the area of two-dimensional shapes, the surface area of the prism changed by the square of the scale factor.

In general, when you dilate any three-dimensional solid by scale factor k, the surface area is multiplied by k^2 and the volume is multiplied by k^3.

Practice

Scaling Solids

1. It takes 2 ounces of paint to completely cover all 6 sides of a rectangular prism box which holds 15 cups of sugar. Double the dimensions of the box. Approximately how much paint would the new box need? How much sugar would it hold?

30 02

2. A solid with volume 12 cubic units is dilated by a scale factor of k. Find the volume of the image for each given value of k.

   a. $k = \frac{1}{4}$ 3

   b. $k = 0.4$ 4.8

   c. $k = 1$ 12

   d. $k = 1.2$ 14.4

   e. $k = \frac{5}{3}$ 20

3. A solid's volume is 10 cubic inches. The solid is dilated by a scale factor of 3.5. Kiran says, "I calculated the volume of the image as 35 cubic inches, but I don't think that's right."

   a. What might Kiran have done wrong?

Kiran might have calculated it incorrectly

   b. What is the volume of the image?

?

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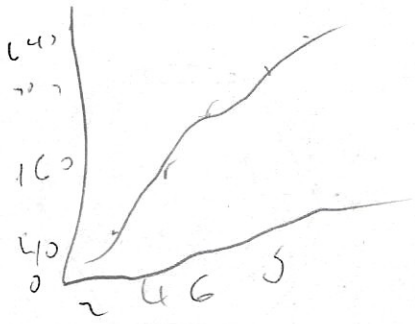
4. A parallelogram has an area of 10 square feet. (Lesson 5-5)

a. Complete the table that shows the relationship between the dilated area (x) and the scale factor (y).

Dilated Area in Square Feet	Scale Factor
0	0
40	2
160	4
360	6
640	8

b. Plot the points in the table on coordinate axes and connect them to create a smooth curve.



5. A figure has an area of 4 square units. The equation $y = \sqrt{\frac{x}{4}}$ represents the scale factor of y by which the solid must be dilated to obtain an image with area x square units. Select **all** points which are on the graph representing this equation. (Lesson 5-5)

(A.) (0, 0)

(B.) $\left(1, \frac{1}{2}\right)$

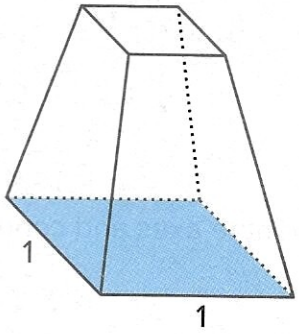
(C.) (1, 1)

(D.) (4, 1)

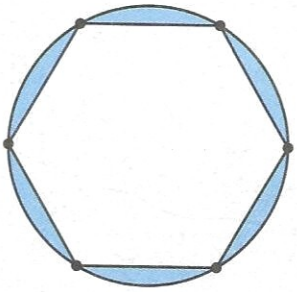
(E.) (8, 2)

6. Tyler is designing a banner that will welcome people to a festival. The design for the banner has an area of 1.5 square feet. The actual banner will be a dilation of the design by a factor of 5. What will the area of the actual banner be? **(Lesson 5-4)**

7. The horizontal cross sections of this figure are dilations of the bottom rectangle using a point above the rectangle as a center and scale factors from $\frac{1}{2}$ to 1. Sketch an example of a cross section that is created from using a scale factor of $\frac{3}{4}$. Label the dimensions of the cross section that you sketch. **(Lesson 5-3)**



8. *Technology required.* A regular hexagon is inscribed in a circle of radius 1 inch. What is the area of the shaded region? **(Lesson 4-10)**



9. Two distinct lines, ℓ and m, are each perpendicular to the same line n. Explain why ℓ and m are parallel lines. **(Lesson 1-6)**

Lesson 5-7

The Root of the Problem

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Learning Goal Let's look at relationships between volumes, areas, and scale factors using graphs and situations.

Warm Up

7.1 The Number That Cubes

A cube whose side lengths measure 1 unit has been dilated by several scale factors to make new cubes.

1. For what scale factor will the volume of the dilated cube be 27 cubic units?

K=3

2. For what scale factor will the volume of the dilated cube be 1,000 cubic units?

K=10

3. Estimate the scale factor that would be needed to make a cube with volume of 1,001 cubic units.

10.003 33222

4. Estimate the scale factor that would be needed to make a cube with volume of 7 cubic units.

1.9129

Activity

7.2 Thinking Inside the Box

A shipping company makes cube-shaped boxes. Their basic box measures 1 foot per side. They want to know how to scale the basic box to build new boxes of various volumes.

1. If the company wants a box with a volume of 8 cubic feet, by what scale factor do they need to dilate the box?

2. If they want a box with a volume of 10 cubic feet, approximately what scale factor do they need?

3. The company decides to create a graph to help analyze the relationship between volume (x) and scale factor (y). Complete the table, rounding values to the nearest hundredth if needed.

   Then, on graph paper, plot the points and connect them with a smooth curve.

Volume in Cubic Feet	Scale Factor
0	
1	
5	
8	
10	
15	
20	
27	

4. The graph shows the relationship between the volume of the dilated box and the scale factor. Write an equation that describes this relationship.

5. Suppose the company builds a box with volume 21 cubic feet, then decides to build another with volume 25 cubic feet. Use your graph to estimate how much the scale factor changes between these 2 dilated boxes.

6. Use your graph to estimate how the scale factor changes between a box with volume of 1 cubic foot and one with volume of 5 cubic feet.

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Activity

7.3 Satellite Scale Factors

A government agency is redesigning a satellite, or an object that goes in orbit around Earth. The surface of the satellite is covered with solar panels that supply the satellite with energy. The interior of the satellite is filled with scientific instruments. In the current design, the satellite has a surface area of 5.4 square feet and a volume of 1.2 cubic feet.

1. If the agency wants to increase the surface area to 21.6 square feet so the satellite can generate more energy, by what scale factor do they need to dilate the satellite?

$4 = k^2$ $k = 2$

2. If the agency instead wants to increase the volume to 4.05 cubic feet to fit in more scientific instruments, by what scale factor do they need to dilate the satellite?

$3.375 = k^3$

Are you ready for more?

Is it possible to dilate the satellite in the activity so that the number of square feet of surface area is equal to the number of cubic feet of volume? Explain or show your reasoning.

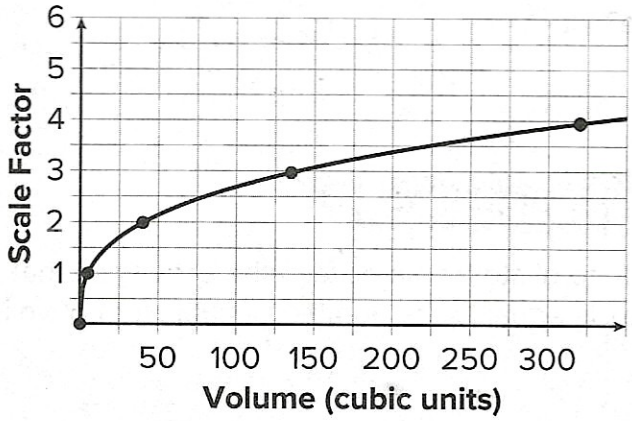
Summary

The Root of the Problem

Suppose a prism has a volume of 5 cubic units. The prism is dilated, and the resulting solid has volume 320 cubic units. If we want to find the scale factor that was used in the dilation, we start by dividing the new volume, 320 cubic units, by the original volume, 5 cubic units, to find that the prism's volume increased by a factor of 64. We know that when a solid is dilated by a scale factor k, the volume is multiplied by k^3. So, we need to find the number whose cube is 64. This number is called the **cube root** of 64 and is written $\sqrt[3]{64}$. We know $\sqrt[3]{64} = 4$ because $4 \cdot 4 \cdot 4 = 64$. That is, the prism was dilated by a scale factor of $k = 4$.

We can create a graph that shows the relationship between the volume, V, of a dilated solid and the scale factor, k, needed to achieve it. Let's use the prism with volume 5 cubic units as an example. Create a table of values, then plot the points and connect them with a smooth curve.

Dilated Volume in Cubic Units (V)	Scale Factor (k)
0	0
5	1
40	2
135	3
320	4



This graph represents the equation $k = \sqrt[3]{\frac{V}{5}}$. The graph rises relatively steeply from (0,0) but quickly flattens out.

We can also find the scale factor of dilation if we know the *surface areas* of the original and dilated solids. Suppose a cylinder has surface area 35 square units, and is dilated resulting in a surface area of 218.75 square units. Divide the numbers to find that the surface area increased by a factor of 6.25. Take the square root of 6.25 to conclude that the solid was dilated by a factor of 2.5.

Glossary

cube root

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Practice

The Root of the Problem

1. A solid with volume 8 cubic units is dilated by a scale factor of k to obtain a solid with volume V cubic units. Find the value of k which results in an image with each given volume.

   a. 216 cubic units b. 1 cubic unit c. 1,000 cubic units

2. A solid has volume 7 cubic units. The equation $k = \sqrt[3]{\frac{V}{7}}$ represents the scale factor of k by which the solid must be dilated to obtain an image with volume V cubic units. Select **all** points which are on the graph representing this equation.

   A. (0, 0) D. (7, 1) G. (56, 2)

   B. (1, 1) E. (14, 2) H. (27, 3)

   C. (1, 7) F. (49, 2)

3. A solid with surface area 8 square units is dilated by a scale factor of k to obtain a solid with surface area A square units. Find the value of k which leads to an image with each given surface area.

   a. 512 square units c. 8 square units

   b. $\frac{1}{2}$ square unit

4. It takes $\frac{1}{8}$ of a roll of wrapping paper to completely cover all 6 sides of a small box that is shaped like a rectangular prism. The box has a volume of 10 cubic inches. Suppose the dimensions of the box are tripled. (Lesson 5-6)

   a. How many rolls of wrapping paper will it take to cover all 6 sides of the new box? It will take the amount of wraping paper

   b. What is the volume of the new box? the volume of the box is?

5. A solid with volume 8 cubic units is dilated by a scale factor of k. Find the volume of the image for each given value of k. (Lesson 5-6)

   a. $k = \frac{1}{2}$

   b. $k = 0.6$

   c. $k = 1$

   d. $k = 1.5$

6. A figure has an area of 9 square units. The equation $y = \sqrt{\frac{x}{9}}$ represents the scale factor of y by which the solid must be dilated to obtain an image with area of x square units. Select **all** points which are on the graph representing this equation. (Lesson 5-5)

   (A.) (0, 0)

   (B.) (1, 1)

   (C.) (1, 3)

   (D.) (3, 1)

   (E.) (9, 1)

   (F.) (9, 3)

   (G.) (18, 2)

   (H.) (36, 2)

7. Noah edits the school newspaper. He is planning to print a photograph of a flyer for the upcoming school play. The original flyer has an area of 576 square inches. The picture Noah prints will be a dilation of the flyer using a scale factor of $\frac{1}{4}$. What will be the area of the picture of the flyer in the newspaper? (Lesson 5-4)

8. Angle S is 90 degrees and angle T is 45 degrees. Side ST is 3 feet. How long is side SU? (Lesson 4-6)

Lesson 5-8

Speaking of Scaling

NAME _____ DATE _____ PERIOD _____

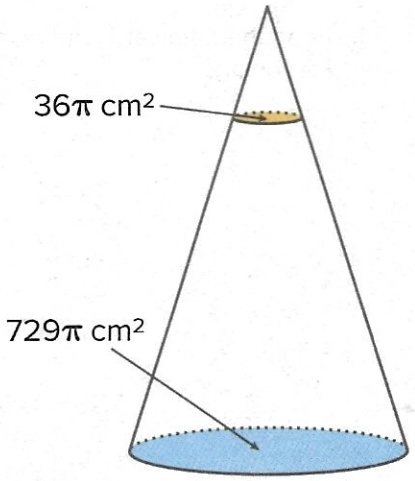
Learning Goal Let's practice moving back and forth between scale factors for lengths, surface areas, and volumes.

Warm Up

8.1 Going Backwards

The image shows a cone that has a base with area 36π square centimeters. The cone has been dilated using the top vertex as a center. The area of the dilated cone's base is 729π square centimeters.

$A' = A \cdot k^2$



What was the scale factor of the dilation?

Activity

8.2 Info Gap: Originals and Dilations

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

Activity

8.3 Jumbo Can

A beverage company manufactures and fills juice cans. They spend $0.04 on materials for each can, and fill each can with $0.27 worth of juice.

The marketing team wants to make a jumbo version of the can that's a dilated version of the original. They can spend at most $0.16 on materials for the new can. There's no restriction on how much they can spend on the juice to fill each can. The team wants to make the new can as large as possible given their budget.

1. By what factor will the height of the can increase? Explain your reasoning.

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2. By what factor will the radius of the can increase? Explain your reasoning.

3. Create drawings of the original and jumbo cans.

4. What geometric solid do the cans resemble? What are some possible differences between the geometric solid and the actual can?

5. What will be the total cost for materials and juice fill for the jumbo can? Explain or show your reasoning.

6. Describe any other factors that might cause the total cost to be different from your answer.

Are you ready for more?

As of 2019, the Burj Khalifa, located in Dubai, was the tallest building in the world. Suppose a scale model of the Burj Khalifa (without antennae) is 30 inches tall.

1. To what scale is this model? You will need to use the internet or another resource to find the actual height of the building.

2. How tall would a model of the Eiffel tower be at this scale?

Summary

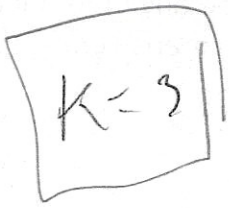
Speaking of Scaling

Suppose a solid is dilated. If we know the factor by which the surface area or volume scale changed, we can work backwards to find the scale factor of dilation. Then we can use that information to solve problems.

A company sells 10 inch by 10 inch by 14 inch 5-gallon aquariums, but a museum wants to buy a 135-gallon aquarium with the same shape. The company needs to know the dimensions of the new tank and by what factor the surface area will change.

Gallons are a measure of volume. So, the volume of the tank increases by a factor of $135 \div 5 = 27$. To find the scale factor for the dimensions of the tank, calculate the cube root of 27, or 3. This tells us that the height, length, and width of the tank will each be multiplied by 3. Next, we can square the scale factor of 3 to find that the tank's surface area will increase by a factor of $3^2 = 9$.

	Original Aquarium	Dilated Aquarium
Height (inches)	10	$10 \cdot 3 = 30$
Length (inches)	14	$14 \cdot 3 = 42$
Width (inches)	10	$10 \cdot 3 = 30$
Surface Area (square inches)	760	$760 \cdot 9 = 6{,}840$
Volume (gallons)	5	$5 \cdot 27 = 135$



gram Publishing

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Practice

Speaking of Scaling

1. A baker makes cupcakes that serve 1 person each. For a party, the baker is asked to make a large cake in the same shape as the cupcake that will serve 100 people.

   a. By what scale factor will the cake need to be dilated?

   1/2

   b. The cakes are contained in decorative paper liners. How many times more paper will be required for the dilated cake as for the original?

   It will take 3 times

2. A board game manufacturer wraps its game boxes in plastic. Its most popular game comes in a box that's 8 cm tall and uses 1408 square centimeters of plastic wrap. The company sells a travel version of the game in a box that's a dilation of the original box. The travel version uses 198 square centimeters of plastic wrap. How tall is the travel version's box?

   the box is 1 ft tall

3. A cone has surface area 360π in^2 and volume 800π in^3. The cone is dilated, and the surface area of the dilated cone is 2, 250π in^2. What is the dilated cone's volume?

   V = 4

4. A scale model of an oil tanker truck can hold 1 gallon of oil. If a trucking company wants the full-size truck to hold 8,000 gallons of oil, what scale factor must they apply to the model? (Lesson 5-7)

5. A solid has volume 4 cubic units. The equation $k = \sqrt[3]{\frac{V}{4}}$ represents the scale factor of k by which the solid must be dilated to obtain an image with volume V cubic units. List 2 points which are on the graph representing this equation. (Lesson 5-7)

6. A cube has edge length 3 inches. (Lesson 5-6)

   a. Find the surface area and volume of the cube.

   b. The cube is dilated by a scale factor of 0.5. Find the surface area and volume of the image.

7. A circle with an area of 100π square centimeters is dilated so that its image has an area of 25π square centimeters. What is the scale factor of the dilation? (Lesson 5-5)

   (A.) 4

   (B.) 2

   (C.) $\frac{1}{2}$

   (D.) $\frac{1}{4}$

Lesson 5-9

Cylinder Volumes

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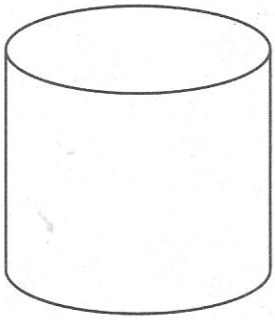
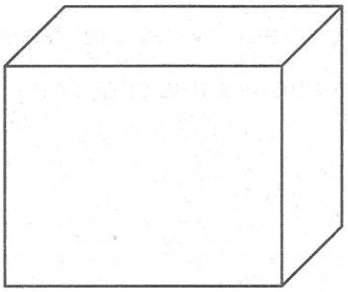
Learning Goal Let's analyze cylinder volumes.

Warm Up

9.1 The Same But Different

Here are two solids.





1. What information would you need to calculate the volume of each solid?

2. What is the same and different about how you would find the volume of each solid?

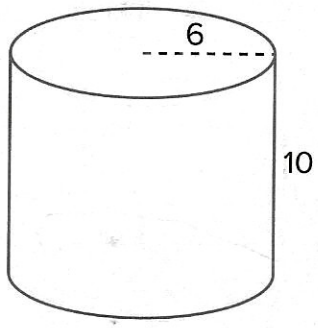
$V = 4\pi \cdot 9 \cdot 1 = ?$ CJT

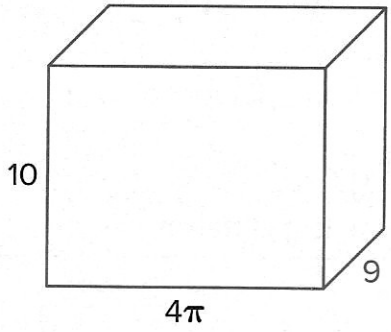
$A = \pi r^2$

Activity

9.2 Water Transfer

Here are two containers. All measurements are in centimeters.





1. Suppose the prism contains water that reaches a height of 1 cm.
   a. Draw a representation of this situation.

   b. The water is poured from the prism into the cylinder. What is the height of the water in the cylinder? Explain your reasoning.

2. Suppose the prism contained water that reached a height of 3 cm instead of 1 cm. If the water were poured into the empty cylinder, what would the height of the water in the cylinder be?

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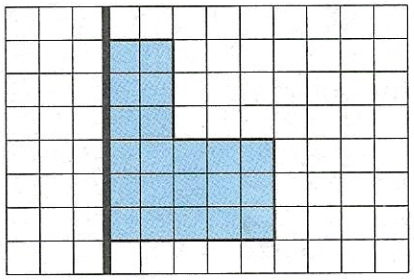
Activity

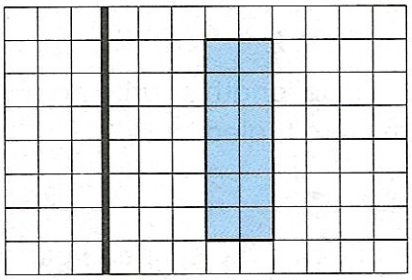
9.3 Revisiting Rotation

Suppose each two-dimensional figure is rotated around the vertical axis shown. Each small square in the grid represents 1 square centimeter.

A

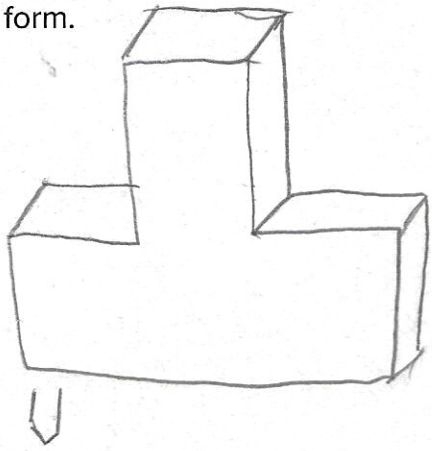


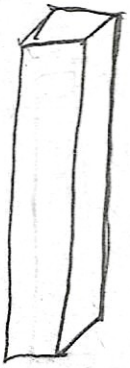
B

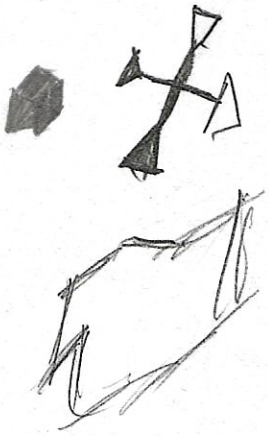


For each solid:

1. Either sketch **or** describe in words the three-dimensional solid that would form.







$V = \pi \cdot 2^2 \cdot 3$

$= \pi 4 \cdot 3 = 12\pi$

2. Find the solid's volume.

Are you ready for more?

1. Given a cylinder with radius r and height h, write an expression for the volume if each of these changes were made.
   a. The height is tripled to $3h$ and the radius remains r.

   b. The radius is tripled to $3r$ and the height remains h.

2. Given a cube of side length s, write an expression for the volume if the side length were tripled to $3s$.

3. Which change affected the shape's volume the most? The least? Explain or show your reasoning.

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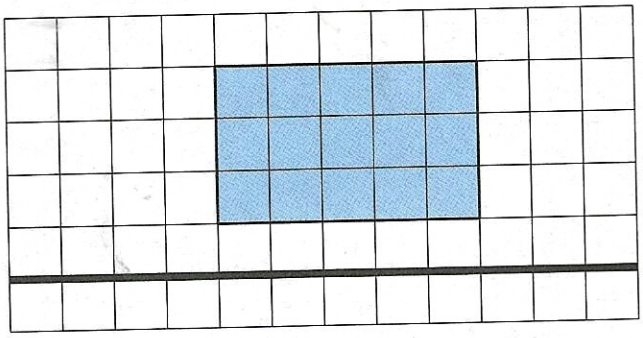
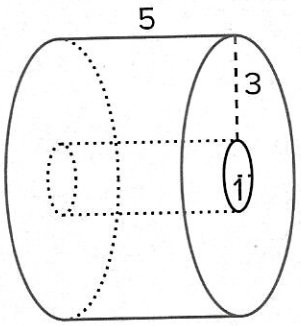
Summary

Cylinder Volumes

Cylinder and prism volumes can be found by multiplying the area of the figure's base by its height. The formula $V = Bh$, where V represents volume, B is the area of the base, and h is height, captures this concept. Consider the solid formed by rotating this rectangle around the horizontal axis shown. The result is a hollow cylinder of height 5 units with inner radius 1 unit and outer radius 4 units.





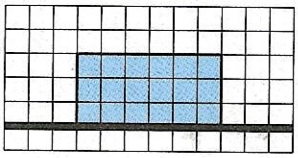
To calculate the volume of the outer cylinder, start by finding the area of the circular base. The circle's radius measures 4 units, so its area is 16π square units because $\pi(4)^2 = 16\pi$. Multiply that by the cylinder's height of 5 units to get 80π cubic units.

For the inner cylinder, the area of the base is π square units, because $\pi(1)^2 = \pi$. The volume is therefore 5π cubic units. Now subtract the volume of the inner, hollow part from the volume of the outer cylinder to get the volume of the solid: 75π cubic units because $80\pi - 5\pi = 75\pi$.

Practice

Cylinder Volumes

1. Each small square represents 1 square centimeter.

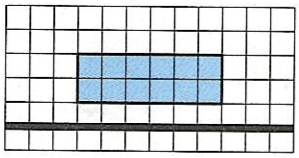


   a. Sketch and label the solid formed by rotating this two-dimensional figure around the horizontal axis shown.

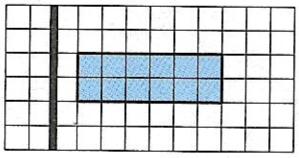
   b. What is the volume of this solid?

2. Match each two-dimensional figure with the solid formed from rotating it using the bold axis shown.

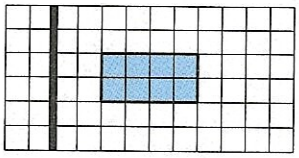
A.



B.

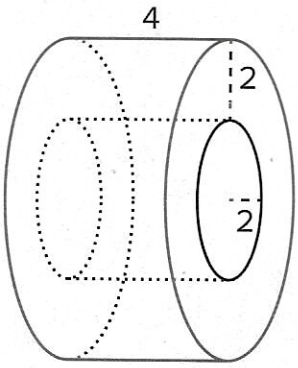


C.

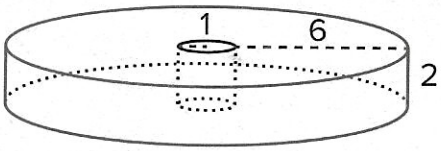


D.

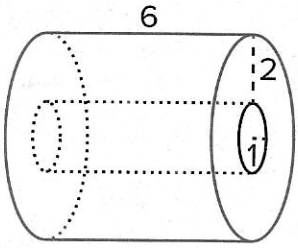
1.



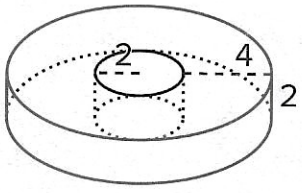
2.



3.



4.



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3. Find the volume of each solid.

   a. a cylinder with radius 4 inches and height 3 inches

   b. a cylinder with radius 3 inches and height 4 inches

   c. a hexagonal prism whose base has area 30.5 square centimeters and whose height is 6.5 centimeters

   d. a prism 5 feet tall whose base is a right triangle with leg lengths 6 feet and 7 feet

4. *Technology required.* A donut shop makes donuts that serve 1 person each. For a party, the baker is asked to make a large donut in the same shape that will serve 25 people. **(Lesson 5-8)**

   a. By what scale factor will the donut need to be dilated? Round your answer to the nearest tenth.

   b. The donuts are coated in a thin layer of glaze. Assume the thickness of the glaze is the same on both donuts. How many times more glaze will be required for the dilated donut as for the original? Round your answer to the nearest tenth.

5. A toothpaste manufacturer wraps its boxes in plastic. Its most popular toothpaste comes in a box that's 8 centimeters tall and uses 1,224 square centimeters of plastic wrap. The company sells a travel version of the toothpaste in a box that's a dilation of the original box. The travel version uses 306 square centimeters of plastic wrap. How tall is the travel version's box? **(Lesson 5-8)**

   (A.) $\frac{1}{2}$ centimeters

   (B.) 2 centimeters

   (C.) 4 centimeters

   (D.) 5 centimeters

6. A solid with volume 32 cubic units is dilated by a scale factor of k to obtain a solid with volume 4 cubic units. Find the value of k. **(Lesson 5-7)**

(A.) 8

(B.) $\frac{1}{8}$

(C.) $\frac{1}{4}$

(D.) $\frac{1}{2}$

7. A cube has edge length 4 inches. **(Lesson 5-6)**

   a. Find the surface area and volume of the cube.

   b. The cube is dilated by a scale factor of 0.25. Find the surface area and volume of the image.

Lesson 5-10

Cross Sections and Volume

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Learning Goal Let's look at how cross sections and volume are related.

Warm Up

10.1 Two Stacks of Coins

The images show the same number of coins arranged in different ways.

A

B

1. How are the two coin stacks different from each other?

2. Does either stack of coins resemble a geometric solid? If so, which stack and what solid?

3. How do the heights of the two stacks compare?

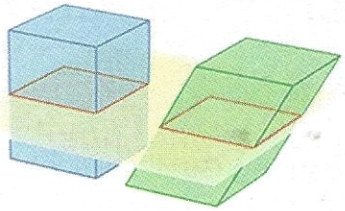
4. How do the volumes of the two stacks compare? Explain your reasoning.

Illustrative Math

Activity

10.2 Rectangular Prism, Shifted

The image shows two rectangular prisms. The bases of the prisms are congruent. Each base has area B square units, and the prisms are the same height. A plane intersects the two prisms parallel to their bases, creating cross sections.



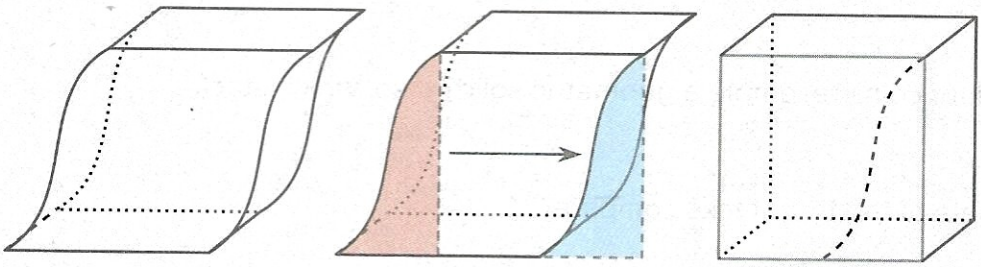
1. Sketch the two cross sections. How do their shapes and areas compare to each other?

2. How would the shape or area of the cross sections change if we moved the plane up or down?

3. How do the volumes of the two prisms compare? Explain your reasoning.

Are you ready for more?

The image shows a recomposition of a solid.



How does this demonstrate Cavalieri's Principle?

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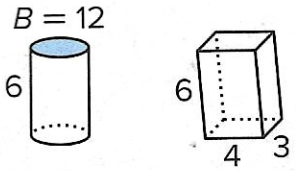
Activity

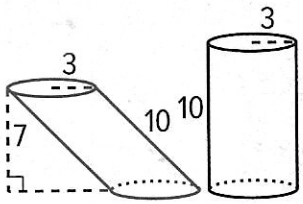
10.3 Equal Volumes?

For each pair of solids, decide whether the volumes of the two solids are equal. Explain your reasoning. If you and your partner disagree, discuss each other's approach until you reach agreement.

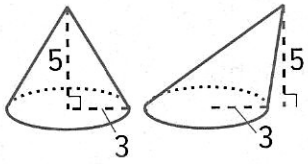
1.



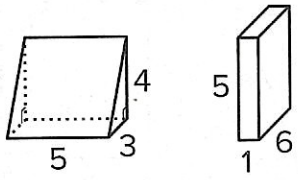
2.



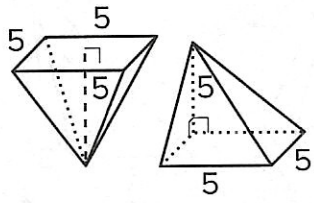
3.



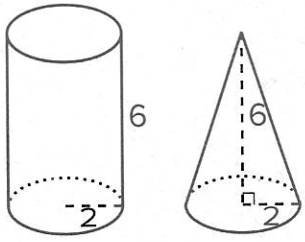
4.



5.



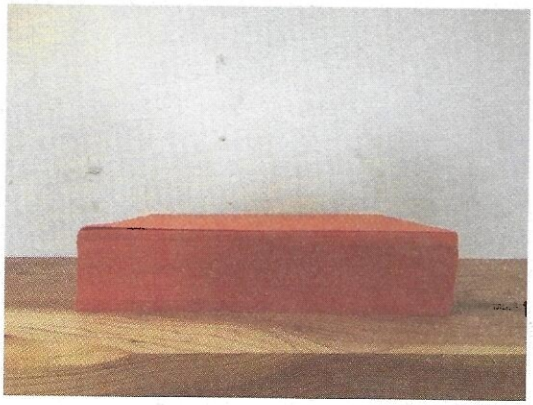
6.

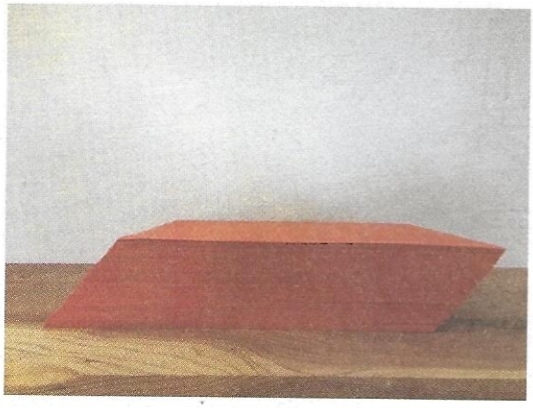


Summary

Cross Sections and Volume

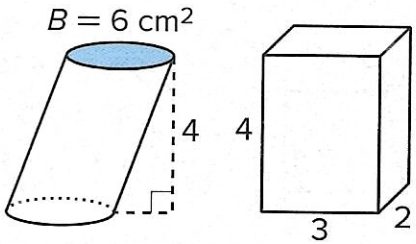
Suppose we have a stack of paper in the shape of a rectangular prism. Then we shift the paper so the prism slants to the side. The first is called a **right** prism because its sides are at right angles to its base. The second one, without right angles between the sides and the base, is called an **oblique** prism. The volume of the prism doesn't change when we shift it—the amount of paper stays the same.



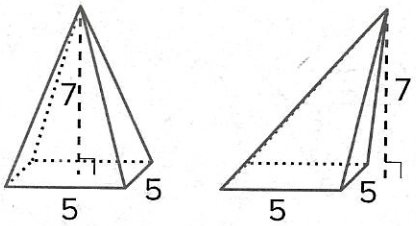


In fact, **Cavalieri's Principle** says that if any two solids are cut into cross sections by parallel planes, and the corresponding cross sections at all heights all have equal areas, then the solids have the same volume.

In this image, all measurements are in centimeters. The oblique cylinder and the right prism have equal volumes because they have the same height and their cross sections at all heights have the same area of 6 square centimeters.



These two pyramids also have equal volumes. Their bases are congruent, and they have the same height. Shifting a solid from right to oblique doesn't change its volume.



Glossary

Cavalieri's Principle
oblique (solid)
right (solid)

Illustrative Math

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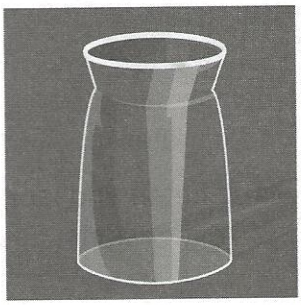
Practice

Cross Sections and Volume

1. This vase has a base of B square inches and a height of h inches. Which statement is true?



   (A.) Its volume is less than Bh cubic inches.

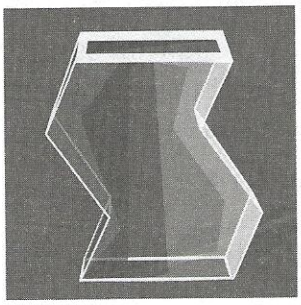
   (B.) Its volume is equal to Bh cubic inches.

   (C.) Its volume is greater than Bh cubic inches.

   (D.) We cannot compare its volume to Bh cubic inches.

2. Noah is deciding which of 2 juice containers he should buy. One container is in the shape of a cylinder with radius 3.5 centimeters and height 12.5 centimeters. The second container is in the shape of a rectangular prism. The prism also has height 12.5 centimeters. Its length is 6.4 centimeters and its width is 4 centimeters. Which juice container has the larger volume?

3. This zigzag crystal vase has a height of 10 inches. The cross sections parallel to the base are always rectangles that are 6 inches wide by 3 inches long.

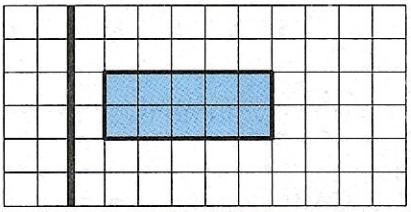


   a. If we assume the crystal itself has no thickness, what would be the volume of the vase?

   b. The crystal is actually half an inch thick on each of the sides and on the bottom. Approximately how much space is contained within the vase? Explain or show your reasoning.

4. List two solids for which the formula $V = Bh$ applies and 2 for which $V = Bh$ does **not** apply. (Lesson 5-9)

5. Each small square represents 1 square centimeter. (Lesson 5-9)



   a. Sketch and label the solid formed by rotating this two-dimensional figure around the vertical axis shown.

   b. What is the volume of this solid?

6. A solid has volume 2 cubic units and surface area 10 square units. The solid is dilated, and the image has volume 128 cubic units. What is the surface area of the image? (Lesson 5-8)

7. A solid with surface area 50 square units is dilated by a scale factor of k to obtain a solid with surface area 200 square units. Find the value of k. (Lesson 5-7)

   (A.) 4

   (B.) 2

   (C.) $\frac{1}{2}$

   (D.) $\frac{1}{4}$

8. Name 2 figures for which all cross sections taken at a particular orientation are congruent. (Lesson 5-2)

Lesson 5-11

Prisms Practice

NAME _____ DATE _____ PERIOD _____

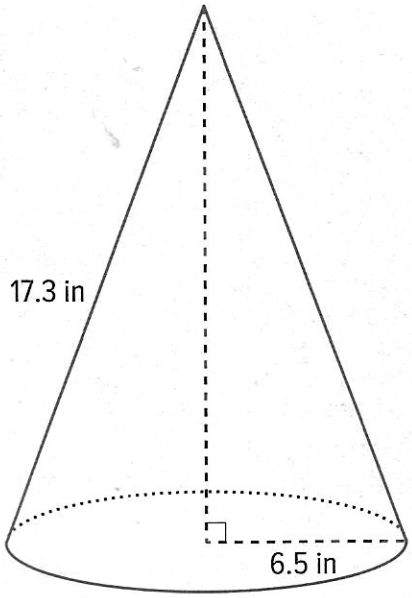
Learning Goal Let's calculate volumes of prisms and cylinders.

Warm Up

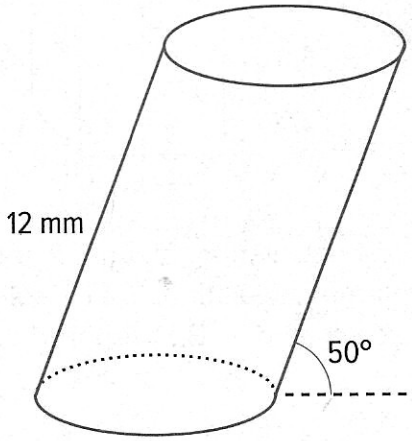
11.1 New Heights

Calculate the height of each solid. Round your answers to the nearest tenth if needed.

A



B

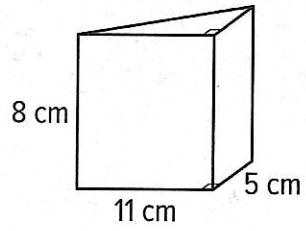


Activity

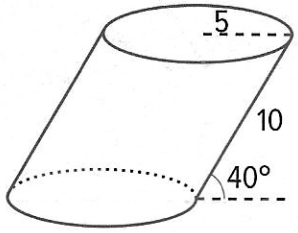
11.2 The Choice is Yours

Here are several solids.

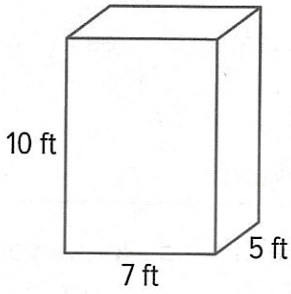
A



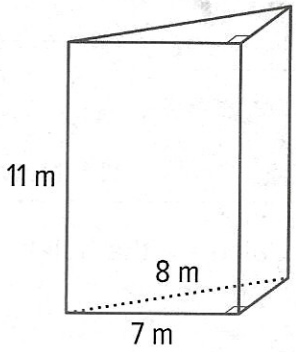
B



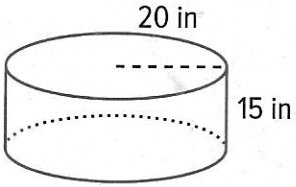
C



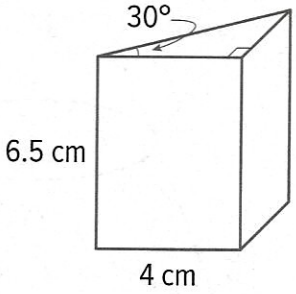
D



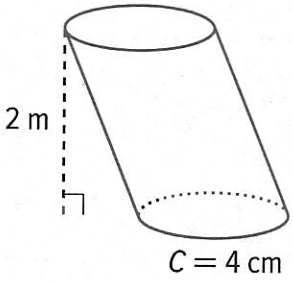
E



F



G



1. Without doing any calculating, identify 2 solids you think would have the least difficult volume calculations and 2 solids that would have the most difficult volume calculations. Be prepared to explain your reasoning.

2. Choose 3 of the solids. At least 1 should be from your "least difficult" list and 1 should be from your "most difficult" list. Calculate the volumes of the solids you chose. Round your answers to the nearest tenth if needed.

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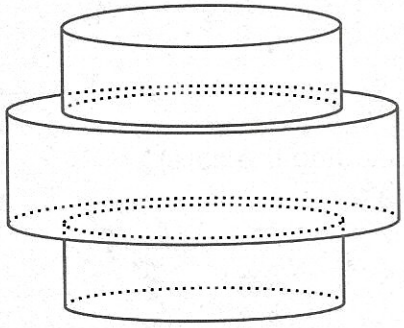
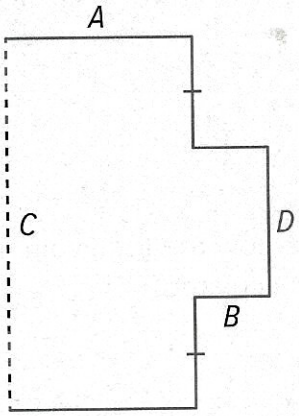
Are you ready for more?

The images show a solid and the two-dimensional figure that was rotated to generate it.





Find 4 different positive integers for the values of A, B, C, and D so that the total volume of the solid is 297π cubic units.

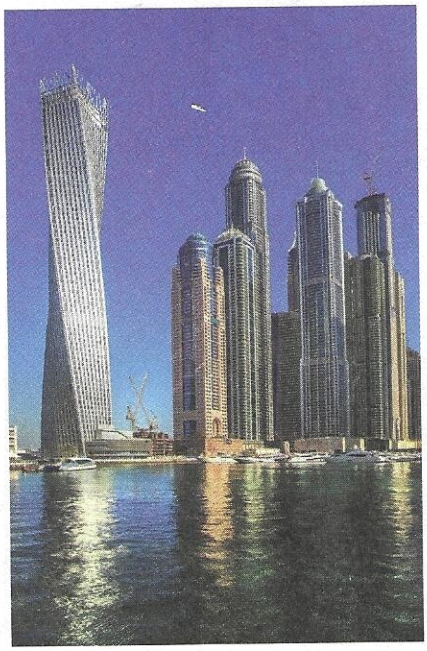
MasterLu/iStock/Getty Images

Activity

11.3 The Cayan Tower

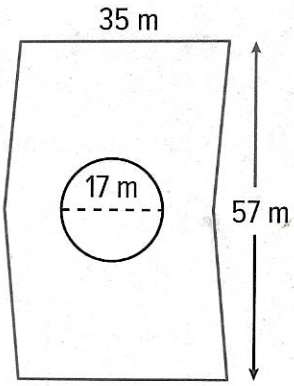
The building on the left side of the picture is the Cayan Tower. It's in the city of Dubai. The tower is about 306 meters tall. It's made up of identical floors that are each rotated slightly compared to the one underneath it.

Each floor is the same chevron shape that is approximately 2 parallelograms put together, with the dimensions shown in the image. The circle in the floor plan shows the cross section of the core, which is used to circulate air and carry pipes and wiring throughout the building.



1. The area of the Cayan Tower's base is 57 • 35 or 1,995 square meters. Why is it possible to find the area of the chevron shape by just multiplying its width and height?

floor plan



2. Describe how the total volume of the building (including the core) can be calculated.

3. What shape is the core of the building, whose cross section is shown in the floor plan as a circle?

4. Describe how the volume of the building's core can be calculated, including describing the measurements that would be used.

5. What percentage of the building's volume is taken up by its core?

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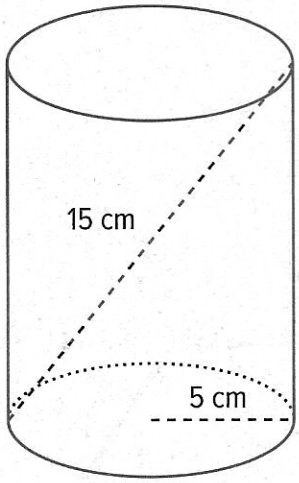
Summary

Prisms Practice

The formula $V = Bh$, where V stands for volume, B is the area of the base, and h is the height, applies to right and oblique cylinders and all prisms. Sometimes, though, additional calculations are needed to find missing measurements before the formula can be applied.

To calculate the volume of this cylinder, first find the area of the base, which is a circle of radius 5 cm. Its area is 25π square centimeters. The cylinder's height isn't given. To find the value of the height, notice that a right triangle is formed by the 15-cm diagonal line, the 10-cm diameter of the circle, and the height of the cylinder. The diagonal line is the triangle's hypotenuse. By the Pythagorean Theorem, $10^2 + h^2 = 15^2$. That means $100 + h^2 = 225$. Subtracting 100 from each side gives $h^2 = 125$, so $h = \sqrt{125}$ centimeters.

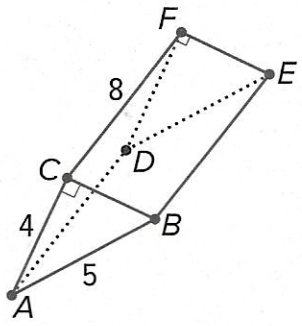


Now, the volume of the cylinder is the area of the base multiplied by the height. This is $25\pi \cdot \sqrt{125}$ cubic centimeters, or approximately 878.1 cubic centimeters.

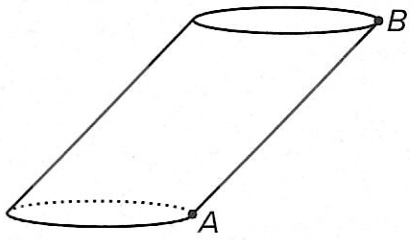
Practice

Prisms Practice

1. This prism has a right triangle for a base. What is the volume of the prism?



2. An oblique cylinder with a base of radius 2 units is shown. The top of the cylinder can be obtained by translating the base by the directed line segment AB which has length $6\sqrt{2}$ units. The segment AB forms a 45° angle with the plane of the base. What is the volume of the cylinder?



3. A prism has a height of 5 inches and a volume of 80 cubic inches. Select **all** figures that could be the base for this prism.

   (A.) a square with side length 4 inches

   (B.) a 2 inch by 8 inch rectangle

   (C.) a circle with radius 2 inches

   (D.) a right triangle with legs 4 inches long

   (E.) a heart-shaped base with an area of 16 square inches

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4. This water bottle has a base with area B square inches and a height of h inches. Tyler thinks the volume of the water bottle is Bh. Elena thinks the volume is less than Bh. **(Lesson 5-10)**

Do you agree with either of them? Explain your reasoning.

Gilmanshin/Shutterstock

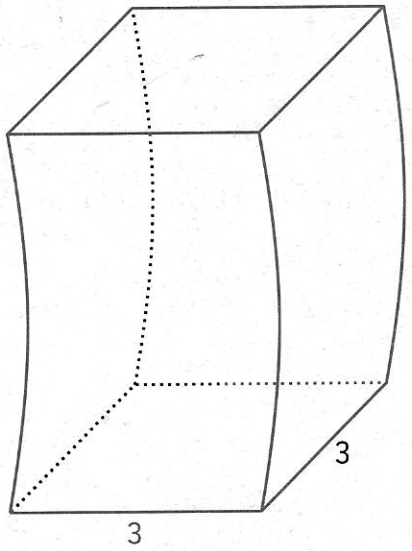
5. This solid has curved sides. All cross sections parallel to the base are squares measuring 3 units on each side. The height from the base to the top is 10 units. What is the volume of this solid? **(Lesson 5-10)**

(A.) 30 cubic units

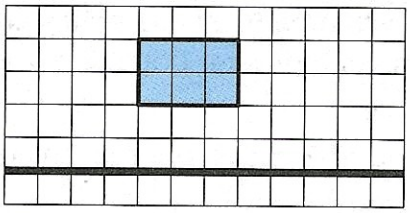
(B.) 60 cubic units

(C.) 90 cubic units

(D.) There is not enough information to determine the volume.



6. Each small square represents 1 square centimeter.
(Lesson 5-9)



a. Sketch and label the solid formed by rotating this two-dimensional figure around the horizontal axis shown.

b. What is the volume of this solid?

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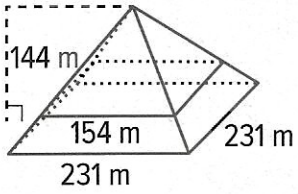
7. A solid has volume 6 cubic units and surface area 22 square units. The solid is dilated, and the image has surface area 198 square units. What is the volume of the image? (Lesson 5-8)

8. The Pyramid of Khufu in Giza, Egypt was the world's tallest free-standing structure for more than 3,500 years. Its original height was about 144 meters. Its base is approximately a square with a side length of 231 meters.

   The diagram shows a cross section created by dilating the base using the top of the pyramid as a center. The cross section has side length of 154 meters. (Lesson 5-3)



   a. What scale factor was used to create the cross section?

   b. What is the height of the cross section?

9. Select **all** statements that **must** be true. (Lesson 2-14)

(A.) The angle bisector of an isosceles triangle is also the perpendicular bisector.

(B.) The angle bisector of any angle divides the angle into 2 congruent parts.

(C.) The median of a triangle is also the perpendicular bisector.

(D.) The median of an isosceles triangles is also the angle bisector.

(E.) The median of a triangle is also the angle bisector.

Lesson 5-12

Prisms and Pyramids

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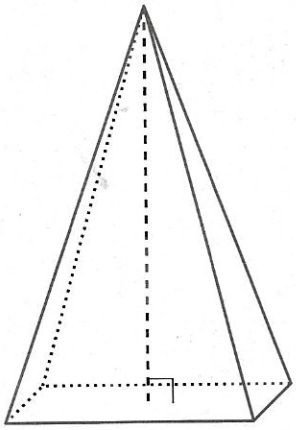
Learning Goal Let's describe relationships between pyramids and prisms.

Warm Up

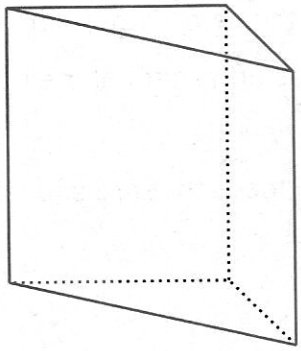
12.1 The Faces of Geometry

Three solids are shown.

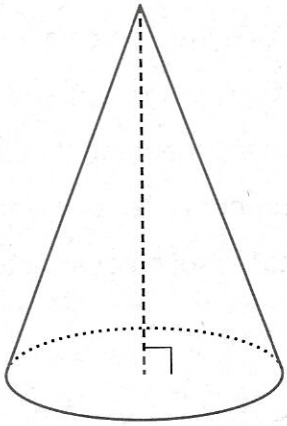
A



B



C



Draw all the surfaces of each solid.

Activity

12.2 Card Sort: Sorting Shapes

Your teacher will give you a set of cards that show geometric solids. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories. Then, sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

Are you ready for more?

The platonic solids are a special group of solids with some specific criteria:

- The faces are all congruent and are all the same regular polygon.
- They are convex, meaning that the faces only meet at their edges.
- The same number of faces meet at every vertex.

1. Draw a platonic solid constructed with faces that are squares.

2. Draw a platonic solid constructed with faces that are triangles.

3. Draw a different platonic solid constructed with faces that are triangles.

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Activity

12.3 Building a Prism from Pyramids

Your teacher will give your group 3 nets. Each student should select 1 of the 3 nets.

1. Cut out your net and assemble a pyramid. The printed side of the net should face outward.

2. Assemble your group's 3 pyramids into a triangular prism. Each pair of triangles with a matching pattern will come together to form one of the rectangular faces of the prism. You will need to disassemble the prism in a later activity, so use only a small amount of tape (or no tape at all if possible).

3. Make a conjecture about the relationship between the volume of the pyramid marked P1 and the volume of the prism.

4. What information would you need to verify that your conjecture is true?

Don't throw away your pyramids! You'll use them again.

Illustrative Math

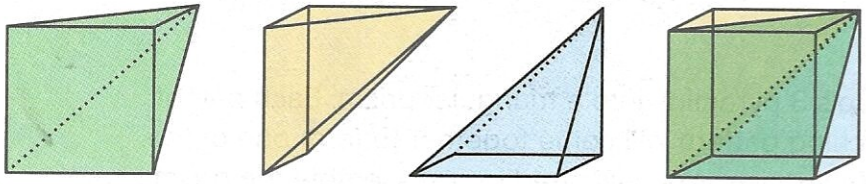
Summary

Prisms and Pyramids

Pyramids and cones are different from prisms and cylinders in that they have just one base and an **apex,** or a single point at which the other faces of the solid meet.

Cones are like cylinders and prisms in that they can be *oblique* or *right*. If a line dropped from the cone's apex at right angles to the base goes through the center of the base, then the cone is right. Otherwise, the cone is oblique. Pyramids that have a clear center in their bases can also be considered right or oblique.

We can use relationships between pyramids and prisms to build a formula for the volume of a pyramid. The image shows 3 square pyramids assembled into a cube. We'll use similar thinking, but with triangular pyramids and prisms, to create a pyramid volume formula in an upcoming lesson.



Glossary

apex

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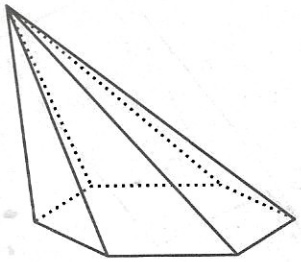
Practice

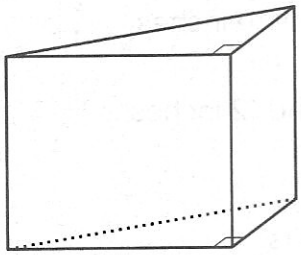
Prisms and Pyramids

1. Give each solid a geometric name. Be as precise as you can.

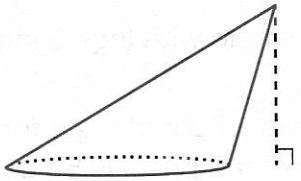
A



B



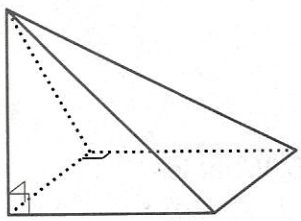
C

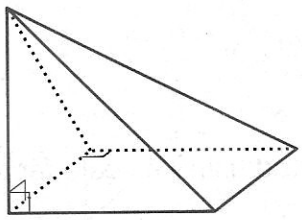


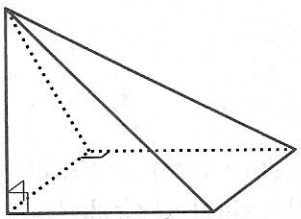
2. Each set of two-dimensional shapes is the complete list of faces from a particular solid. Match each set of shapes with the solid they came from.

A. 2 congruent triangles and 3 rectangles	**1.** right triangular prism
B. 4 triangles and 1 rectangle	**2.** oblique square prism
C. 2 squares and 4 congruent parallelograms	**3.** right triangular pyramid
D. 4 congruent, equilateral triangles	**4.** rectangular pyramid

3. These 3 congruent square pyramids can be assembled into a cube with side length 1 foot. What is the volume of each pyramid?







4. A prism has a height of 4 inches and a volume of 120 cubic inches. Select **all** figures that could be the base for this prism. (Lesson 5-11)

   (A.) a 5 inch by 6 inch rectangle

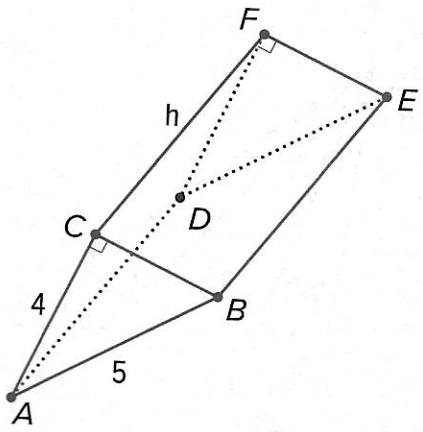
   (B.) a square with side length 5 inches

   (C.) a circle with radius 5 inches

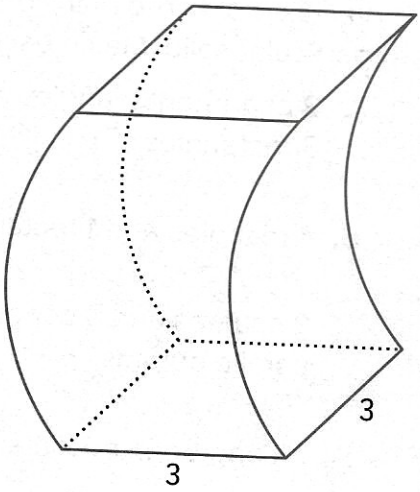
   (D.) a star-shaped base with area 30 square inches

   (E.) a right triangle with legs 5 inches and 12 inches

5. This prism has a right triangle for a base. The volume of the prism is 54 cubic units. What is the value of h? (Lesson 5-11)



6. This solid has curved sides. All cross sections parallel to the base are squares measuring 3 units on each side. The height from the base to the top is 5 units. What is the volume of this solid? (Lesson 5-10)



7. Find the volume of each solid. (Lesson 5-9)

   a. a cylinder with radius 3 inches and height 2 inches

   b. a hexagonal prism whose base has area 4.5 square centimeters and whose height is 7 centimeters

   c. a prism 5 feet tall whose base is a right triangle with leg lengths $\frac{3}{2}$ feet and 9 feet

8. A circle with area π square units is dilated using a scale factor of 5. What is the area of the dilated circle? (Lesson 5-4)

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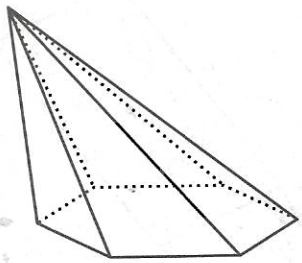
Practice

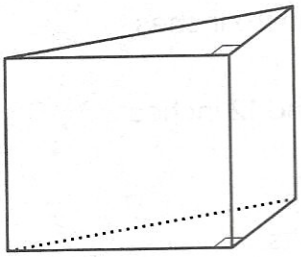
Prisms and Pyramids

1. Give each solid a geometric name. Be as precise as you can.

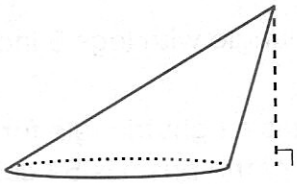
A



B



C



2. Each set of two-dimensional shapes is the complete list of faces from a particular solid. Match each set of shapes with the solid they came from.

A. 2 congruent triangles and 3 rectangles

B. 4 triangles and 1 rectangle

C. 2 squares and 4 congruent parallelograms

D. 4 congruent, equilateral triangles

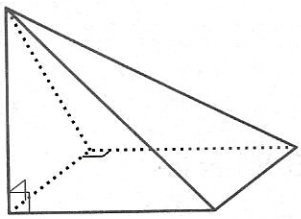
1. right triangular prism

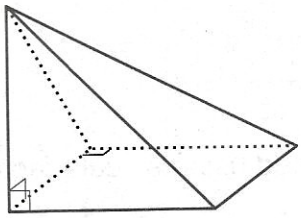
2. oblique square prism

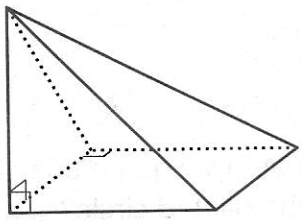
3. right triangular pyramid

4. rectangular pyramid

3. These 3 congruent square pyramids can be assembled into a cube with side length 1 foot. What is the volume of each pyramid?







4. A prism has a height of 4 inches and a volume of 120 cubic inches. Select **all** figures that could be the base for this prism. (Lesson 5-11)

   (A.) a 5 inch by 6 inch rectangle

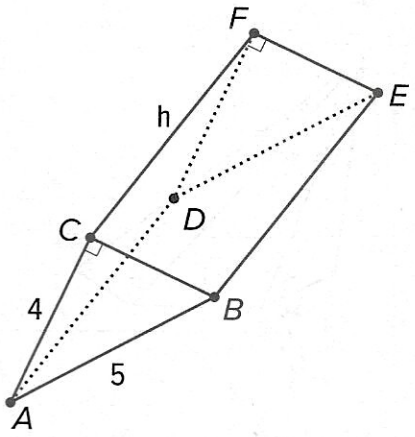
   (B.) a square with side length 5 inches

   (C.) a circle with radius 5 inches

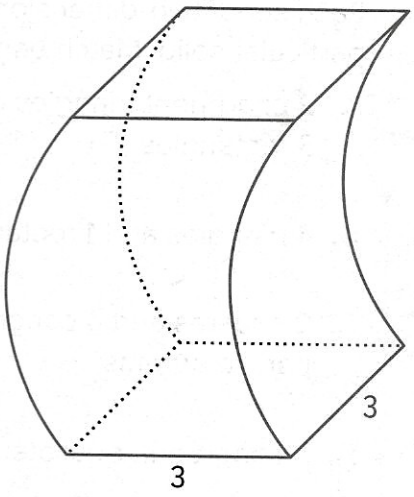
   (D.) a star-shaped base with area 30 square inches

   (E.) a right triangle with legs 5 inches and 12 inches

5. This prism has a right triangle for a base. The volume of the prism is 54 cubic units. What is the value of h? (Lesson 5-11)



6. This solid has curved sides. All cross sections parallel to the base are squares measuring 3 units on each side. The height from the base to the top is 5 units. What is the volume of this solid? (Lesson 5-10)



7. Find the volume of each solid. (Lesson 5-9)

   a. a cylinder with radius 3 inches and height 2 inches

   b. a hexagonal prism whose base has area 4.5 square centimeters and whose height is 7 centimeters

   c. a prism 5 feet tall whose base is a right triangle with leg lengths $\frac{3}{2}$ feet and 9 feet

8. A circle with area π square units is dilated using a scale factor of 5. What is the area of the dilated circle? (Lesson 5-4)

Lesson 5-13

Building a Volume Formula for a Pyramid

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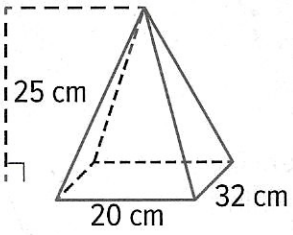
Learning Goal Let's create a formula for the volume of any pyramid or cone.

Warm Up

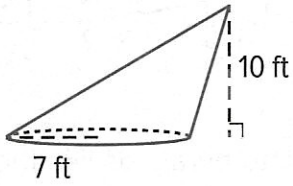
13.1 Cover Your Bases

Two solids are shown.

A



B

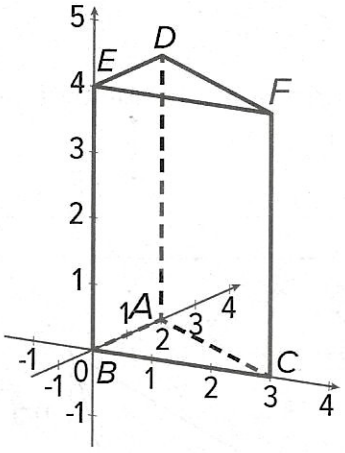


For each solid, draw and label a prism or cylinder that has a base congruent to the solid's and a height equal to the solid's.

Activity

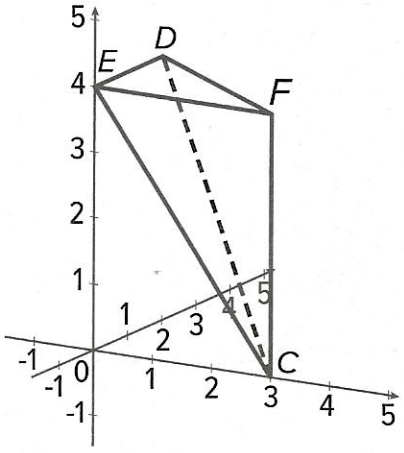
13.2 Splitting a Prism into Pyramids

Here is a triangular prism.

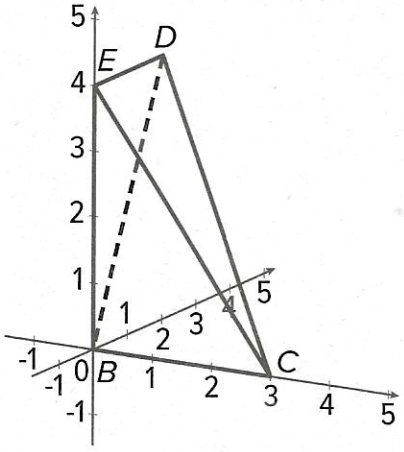


Suppose we split the prism into pyramids like the ones you built earlier. The first pyramid is split off by slicing through points *E*, *D*, and *C*. The remaining part of the prism is sliced through *B*, *C*, and *D*.

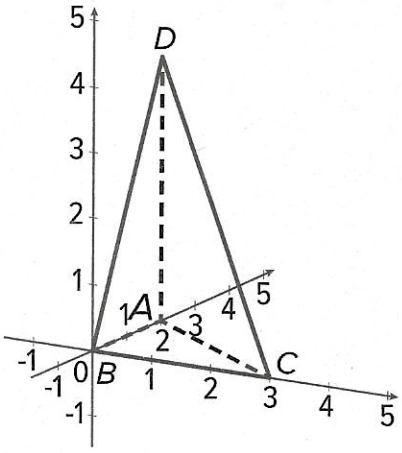
P1



P2



P3



1. Using the pyramids you built, compare pyramids P1 and P3.

   a. Think of the faces marked P1 and P3 as the bases of the pyramids. These triangles are the two bases of the original prism. How do the areas of these two bases compare?

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b. How do the heights of pyramids P1 and P3 compare? Explain your reasoning.

c. How do the volumes of pyramids P1 and P3 compare? Explain your reasoning.

2. Using the pyramids you built, compare pyramids P2 and P3.

a. Think of the gray shaded triangles as the bases of the pyramids. These are formed by slicing one of the prism's rectangular faces down its diagonal. How do the areas of these two bases compare?

b. The heights of pyramids P2 and P3 are equal because when assembled into the prism, the height lines coincide along the length of the prism. How, then, do the volumes of these pyramids compare? Explain your reasoning.

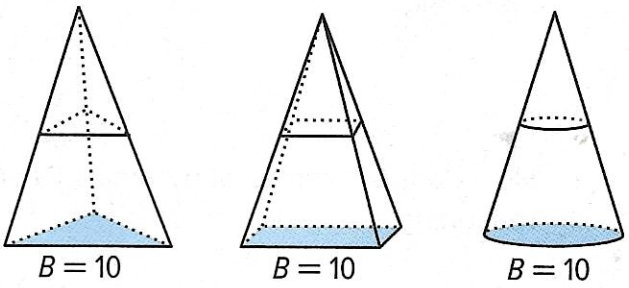
3. Based on your answers, how does the volume of each pyramid compare to the volume of the prism?

4. How could you use this information to find the volume of one of the pyramids?

Activity

13.3 Comparing Cross Sections

Each solid in the image has height 6 units. The area of each solid's base is 10 square units. A cross section has been created in each by dilating the base using the apex as a center with scale factor $k = 0.5$.



1. Calculate the area of each of the 3 cross sections.

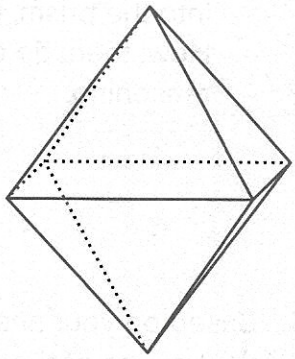
2. Suppose a new cross section was created in each solid, all at the same height, using some scale factor k. How would the areas of these 3 cross sections compare? Explain your reasoning.

3. What does this information about cross sections tell you about the volumes of the 3 solids?

4. Calculate the volume of each of the solids.

Are you ready for more?

An octahedron is a solid whose faces consist of 8 equilateral triangles. Find the volume of an octahedron with edge length ℓ.



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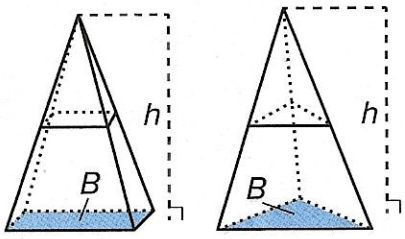
Summary

Building a Volume Formula for a Pyramid

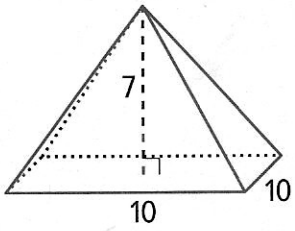
In an earlier activity, we conjectured that a triangular pyramid has one-third the volume of the prism that has the same height and a base congruent to the pyramid's. This turns out to be true.

Suppose we have a pyramid that doesn't have a triangular base. Call the area of its base B and its height h. To understand how to find the volume of such a solid, think about a triangular pyramid that also has an area of B square units and a height of h.

Now find a cross section of each solid by dilating the solid's base using the apex as a center with some scale factor k between 0 and 1. The area of the cross section in both solids will be Bk^2. This is true for any value of k. Since the cross sections at all heights have equal area, the solids have the same volume. The same idea would apply to a cone. This means the expression $\frac{1}{3}Bh$ gives the volume of *any* pyramid or cone.



For example, this pyramid's base has an area of 100 square units. The volume of the pyramid is about 233.3 cubic units, because $\frac{1}{3} \cdot 100 \cdot 7 \approx 233.3$.



Practice

Building a Volume Formula for a Pyramid

1. Find the volume of a pyramid whose base is a square with side lengths of 6 units and height of 8 units.

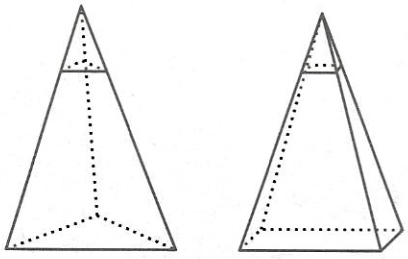
2. A cylinder has radius 9 inches and height 15 inches. A cone has the same radius and height.

   a. Find the volume of the cylinder.

   b. Find the volume of the cone.

   c. What fraction of the cylinder's volume is the cone's volume?

3. Each solid in the image has height 4 units. The area of each solid's base is 8 square units. A cross section has been created in each by dilating the base using the apex as a center with scale factor $k = 0.25$.



   a. Calculate the area of each of the 2 cross sections.

   b. Suppose a new cross section was created in each solid, both at the same height, using some scale factor k. How would the areas of these 2 cross sections compare? Explain your reasoning.

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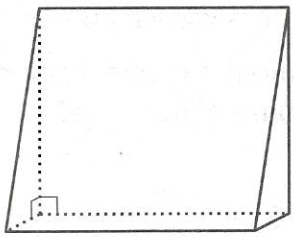
4. Select the most specific and accurate name for the solid in the image. (Lesson 5-12)



(A.) triangular pyramid

(B.) regular prism

(C.) square prism

(D.) right triangular prism

5. A solid can be constructed with 4 triangles and 1 rectangle. What is the name for this solid? (Lesson 5-12)

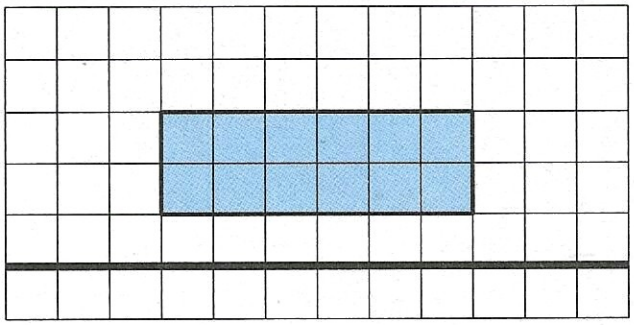
(A.) rectangular pyramid

(B.) triangular pyramid

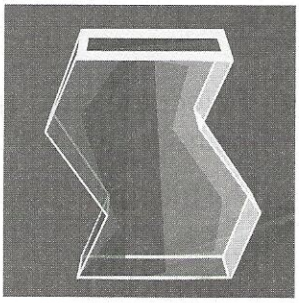
(C.) right triangular prism

(D.) rectangular prism

6. Find the volume of the solid produced by rotating this two-dimensional shape using the axis shown. (Lesson 5-11)



7. This zigzag crystal vase has a height of 20 centimeters. The cross sections parallel to the base are always rectangles that are 12 centimeters wide by 6 centimeters long. (Lesson 5-10)



   a. If we assume the crystal itself has no thickness, what would be the volume of the vase?

   b. The crystal is actually 1 centimeter thick on each of the sides and on the bottom. Approximately how much space is contained within the vase? Explain or show your reasoning.

8. A trapezoid has an area of 10 square units. What scale factor would be required to dilate the trapezoid to have an area of 90 square units? (Lesson 5-5)

   (A) 9

   (B) 6

   (C) 3

   (D) $\frac{1}{3}$

Lesson 5-14

Working with Pyramids

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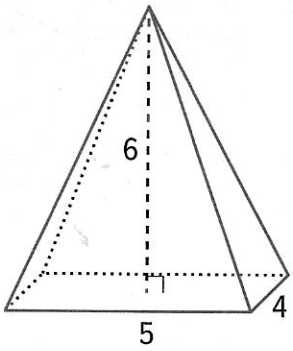
Learning Goal Let's use the pyramid volume formula to solve problems.

Warm Up

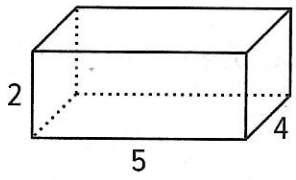
14.1 Volume Matching

Here is a pyramid.

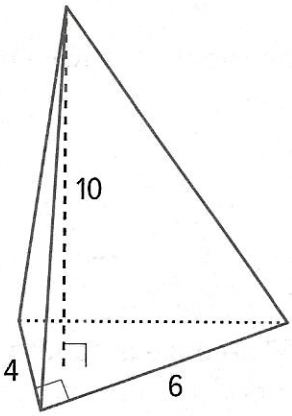


Which, if either, of these solids has the same volume as the pyramid?

A



B

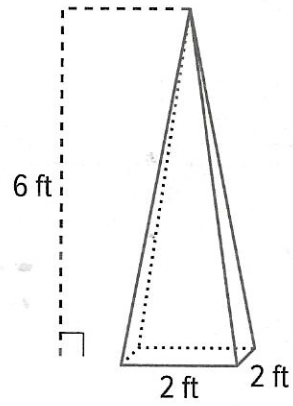


Activity

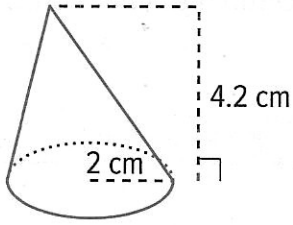
14.2 Practice with Pyramids

1. Calculate the volume of each solid. Round your answers to the nearest tenth if necessary.

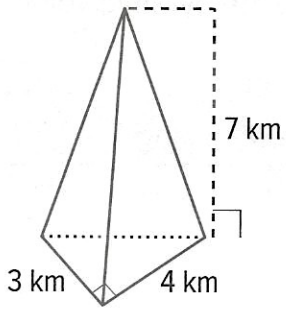
A



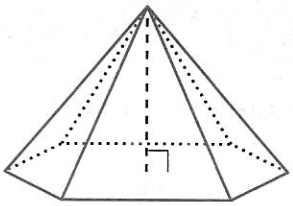
B



C



D: height 12 cm; area of base 32 cm²



2. A particular cone has radius r and height h.
   a. Write an expression for the volume of this cone in terms of r and h.

   b. What is the height of a cone whose volume is 16π cubic units and whose radius is 3 units?

   c. What is the radius of a cone whose volume is 16π cubic units and whose height is 3 units?

Are you ready for more?

The Pyramid of Giza is 455 feet tall. The base is square with a 756-foot side length. How many Olympic-size swimming pool volumes of water can fit inside the Pyramid of Giza? Explain or show your reasoning.

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Activity

14.3 A Chocolate Pyramid

A baker is making a pyramid out of chocolate for a special display in a store. The baker has 20 pounds of chocolate to use, which is about 720 cubic inches. The pyramid must be 15 inches tall. The baker needs to decide how large to make the base, which can be any shape. Draw and label the dimensions of 2 different bases that would work.

Summary

Working with Pyramids

We can work backward from a given volume to find possible dimensions for a cone or pyramid.

Suppose we want to find dimensions for a cone so it has a volume of 900π cubic inches. Start by substituting the volume into the pyramid volume formula to get $900\pi = \frac{1}{3}Bh$. The base of a cone is a circle, so we can write $900\pi = \frac{1}{3}\pi r^2 h$. Multiply both sides of the equation by 3 and divide both sides by π to get $2{,}700 = r^2 h$.

Now consider different possible values for r and h. If we can find a perfect square that divides evenly into 2,700, we can set the square root of that number to be the radius. The number 25 is a perfect square and divdes into 2,700, so choose $r = 5$. Now $2{,}700 = 25h$. This tells us that if the pyramid's radius is 5 inches, its height is 108 inches because $2{,}700 \div 25 = 108$.

These aren't the only possible values. Suppose we set the radius to be 20 inches. Substitute this into the original equation and rearrange to find the value of h.

$$900\pi = \frac{1}{3}\pi(20)^2 h$$

$$900\pi = \frac{1}{3}\pi \cdot 400\text{h}$$

$$2{,}700 = 400\text{h}$$

$$6.75 = h$$

A height of 6.75 inches together with a radius of 20 inches gives the cone a volume of 900π cubic inches.

Practice

Working with Pyramids

1. A pyramid has a height of 5 inches and a volume of 60 cubic inches. Select **all** figures that could be the base for this pyramid.

   (A.) a square with side length 6 inches

   (B.) a 3 inch by 4 inch rectangle

   (C.) a 4 inch by 9 inch rectangle

   (D.) a circle with radius 4 inches

   (E.) a right triangle with one side 5 inches and the hypotenuse 13 inches

   (F.) a hexagon with an area of 36 square inches

2. A company makes a block of cheese in the shape of a rectangular prism with dimensions 4 inches by 2 inches by 2 inches. They want to make a new block, in the shape of a rectangular pyramid, that uses the same amount of cheese. Determine two sets of possible dimensions for the pyramid.

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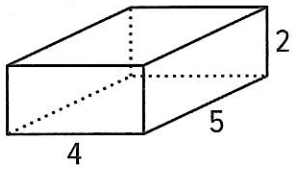
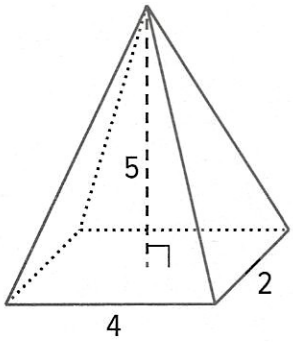
3. Select **all** the solids with volume 40 cubic units.

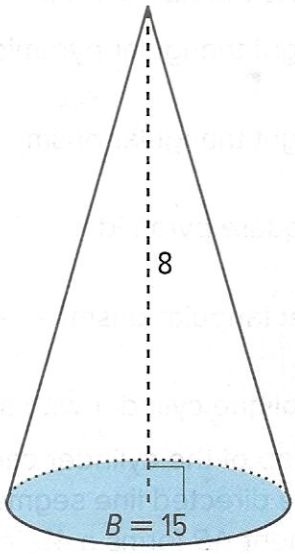
A. rectangular prism



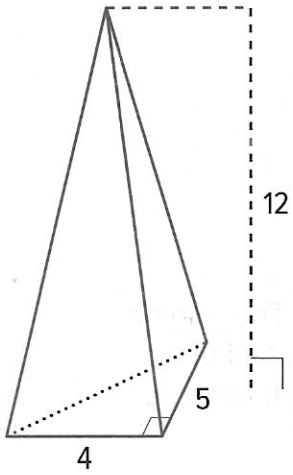
B. right rectangular pyramid



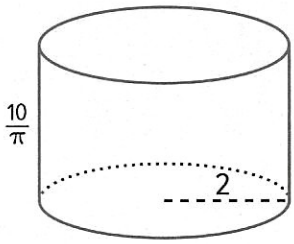
C. right cone



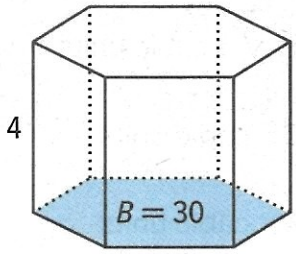
D. triangular pyramid



E. right cylinder



F. right hexagonal prism



(A.) Solid A

(B.) Solid B

(C.) Solid C

(D.) Solid D

(E.) Solid E

(F.) Solid F

4. The volume of a pyramid is 50 cubic units. The base is a square with sides of length 5. What is the height? (Lesson 5-13)

(A.) 2 units

(B.) 4 units

(C.) 6 units

(D.) 10 units

5. A cone and a cylinder have the same radius and height. The volume of the cone is 100π cubic feet. What is the volume of the cylinder? (Lesson 5-13)

6. A solid can be constructed with 2 congruent triangles and 3 rectangles. What is the name of this solid? (Lesson 5-12)

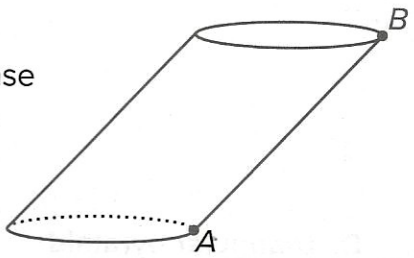
   (A.) right triangular pyramid

   (B.) right triangular prism

   (C.) square pyramid

   (D.) rectangular prism

7. An oblique cylinder with a base of radius 3 is shown.

   The top of the cylinder can be obtained by translating the base by the directed line segment AB which has length $6\sqrt{2}$. The segment AB forms a 45° angle with the plane of the base. What is the volume of the cylinder? (Lesson 5-11)

   (A.) 18π cubic units

   (B.) $18\pi\sqrt{2}$ cubic units

   (C.) 36π cubic units

   (D.) 54π cubic units

8. Mai, Andre, and Lin are discussing the volume of a scaled box. The original box holds 7 cubic inches. The new box holds 448 cubic inches. Mai thinks the scale factor is 4, Andre thinks the scale factor is 16, and Lin thinks the scale factor is 64. Do you agree with any of them? Explain your reasoning. (Lesson 5-6)

Lesson 5-15

Putting All the Solids Together

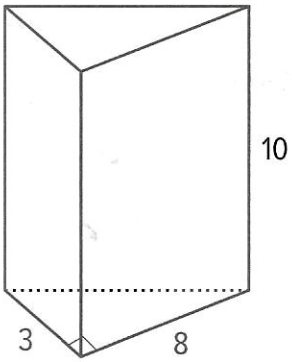
NAME _____ DATE _____ PERIOD _____

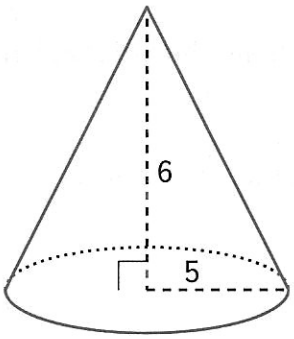
Learning Goal Let's calculate volumes of prisms, cylinders, cones, and pyramids.

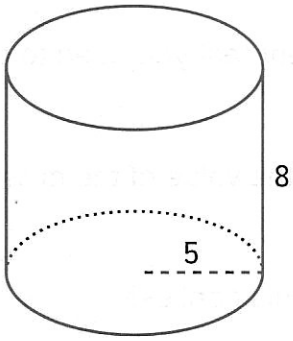
Warm Up

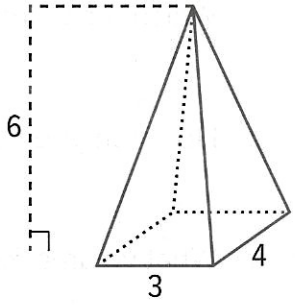
15.1 Math Talk: Volumes

Evaluate the volume of each solid mentally.







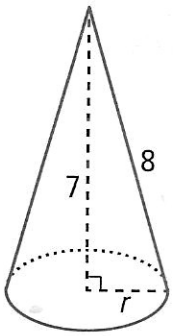


Activity

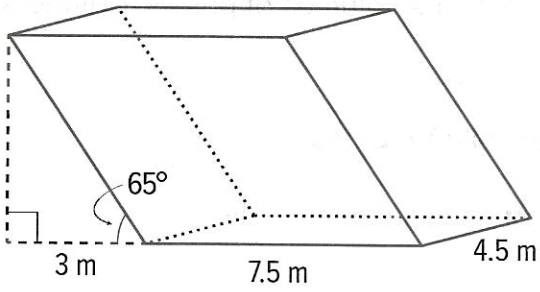
15.2 Missing Measurements

1. Answer the questions for each of the two solids shown.

A



B



a. Which measurement that you need to calculate the volume isn't given?

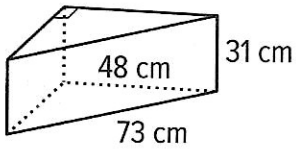
b. How can you find the value of the missing measurement?

c. What volume formula applies?

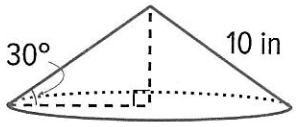
d. Calculate the volume of the solid, rounding to the nearest tenth if necessary.

2. Calculate the volume of each solid, rounding to the nearest tenth if necessary.

A



B



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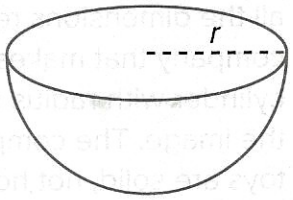
Are you ready for more?

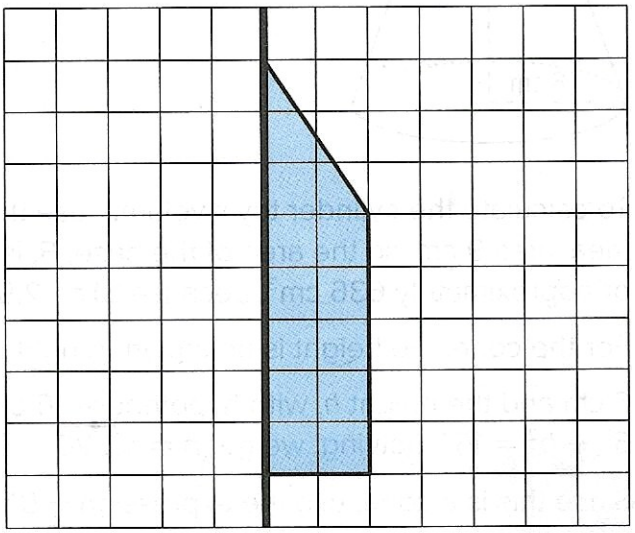
For a sphere with radius r, its volume is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$. Here is a half-sphere bowl pressed out of a piece of sheet metal with area 1 square foot. What is the volume of the bowl?



Activity

15.3 Spinning into Three Dimensions

Suppose this two-dimensional figure is rotated 360 degrees using the vertical axis shown. Each small square on the grid represents 1 square inch.



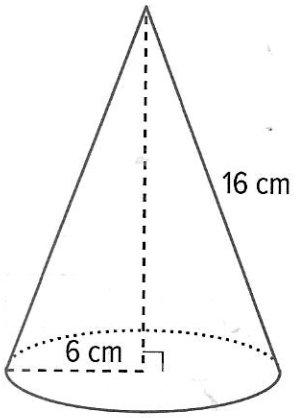
1. Draw the solid that would be traced out. Label the dimensions of the solid.

2. Find the volume of the solid. Round your answer to the nearest tenth if needed.

Summary

Putting All the Solids Together

Before computing volume, it's important to select the right formula and find all the dimensions represented in the formula. For example, consider a company that makes two chew toys for dogs. One toy is in the shape of a cylinder with radius 9 cm and height 2.5 cm. The other looks like the cone in the image. The company wants to know which toy uses more material. The toys are solid, not hollow.



To calculate the cylinder toy's volume, use the expression Bh. The radius measures 9 cm, so the area of the base, B, is 81π cm^2. The volume is 202.5π, or approximately 636 cm^3, because $81\pi \cdot 2.5 = 202.5\pi$.

For the cone, the height is unknown. A right triangle is formed by the radius 6 cm and the height h, with hypotenuse 16 cm. By the Pythagorean Theorem, $6^2 + h^2 = 16^2$. Solving, we get $h = \sqrt{220}$.

Since this is a cone, use the expression $\frac{1}{3}Bh$. The area of the base, B, is 36π cm^2. The volume is approximately 559 cm^3 because $\frac{1}{3} \cdot 36\pi \cdot \sqrt{220} \approx 559$. The cylinder-shaped toy uses more material.

NAME _____ DATE _____ PERIOD _____

Practice

Putting All the Solids Together

1. A right cone has a base with diameter 10 units. The volume of the cone is 100π cubic units. What is the length of a segment drawn from the apex to the edge of the circular base?

2. A right pyramid has a square base with sides of length 10 units. Each segment connecting the apex to a midpoint of a side of the base has length 13 units. What is the volume of the pyramid?

3. For each pair of solids, determine if their volumes are the same or different. If the volumes are different, identify the solid with the greatest volume. Explain your reasoning.

   a. A prism and a pyramid have the same height. The pyramid's base has 3 times the area of the prism's base.

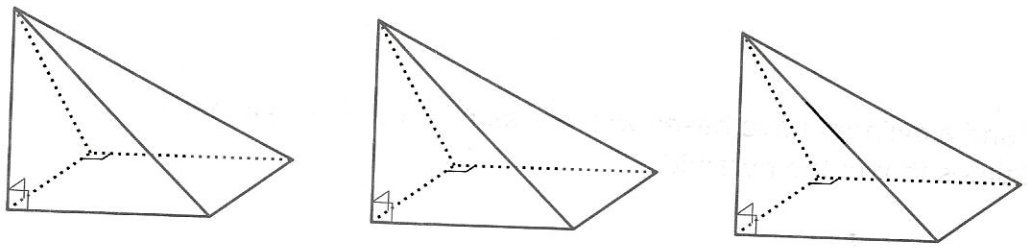
   b. A pyramid and a cylinder have bases with the same area. The cylinder's height is 3 times that of the pyramid.

   c. A cone and a cylinder have the same height. The cone's radius is 3 times the length of the cylinder's radius.

4. A pyramid has a height of 8 inches and a volume of 120 cubic inches. Determine 2 possible shapes, with dimensions, for the base. (Lesson 5-14)

5. A company makes a chocolate bar in the shape of a rectangular prism with dimensions 6 inches by 1 inch by $\frac{1}{2}$ inch. They want to make a new chocolate bar, in the shape of a rectangular pyramid, that uses the same amount of chocolate. Determine two sets of possible dimensions for the pyramid. (Lesson 5-14)

6. These 3 congruent square pyramids can be assembled into a cube with side length 2 feet. What is the volume of each pyramid? (Lesson 5-12)



7. A monster truck wheel has an area of 324π square inches. A toy company wants to create a scaled copy of the monster truck with a wheel area of 9π square inches. What scale factor should the company use? (Lesson 5-7)

Lesson 5-16

Surface Area and Volume

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's use volume and surface area to solve problems.

Warm Up

16.1 Maximize Area

The zoo wants to give the elephants as much space as possible in a rectangular enclosure meant for feeding. The zoo has 180 feet of fencing. What should the dimensions of the rectangle be? Be prepared to share your reasoning.

Sergey Uryadnikov/Shutterstock.com

Activity

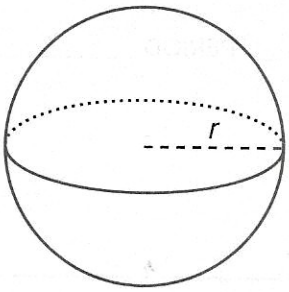
16.2 Maximize in Three Dimensions

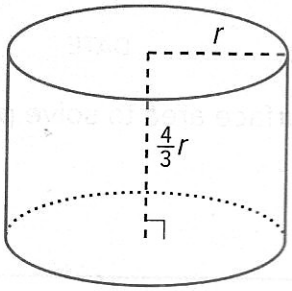
1. Find a set of dimensions for a rectangular prism with volume 60 cubic units. Calculate the surface area of your prism. Add your data to the class chart.

2. A lithium ion battery contains a rectangular prism made of lithium. The energy in the battery is proportional to the surface area of this prism. Assume the lithium has a fixed volume of 60 cubic millimeters. Find the dimensions of a rectangular prism with this volume that maximizes its surface area. What is its surface area?

Are you ready for more?

Minimizing surface area plays a large role in manufacturing. Companies try to use the smallest amount of resources possible to package products in order to save money.





1. Calculate the surface area of the 2 figures shown, which both enclose the same volume.

2. Which container would you recommend a company use to package small candies? Explain your reasoning

Activity

16.3 Assume a Spherical Elephant

For a sphere with radius r, its volume is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.

1. Let's model an elephant with a sphere that has a radius of 4.5 feet.
   a. What is the volume of the elephant?

   b. What is the surface area of the elephant?

2. Let's model a snake with a cylinder of length 3 feet and diameter 0.2 feet.
   a. What is the volume of the snake?

   b. What is the surface area of the snake?

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3. Compute the surface area to volume ratio, or $\frac{SA}{V}$, for each animal.

Activity

16.4 Measuring Strength

Suppose a human is a sphere with a radius of 1 unit, an ant is a sphere with a radius of $\frac{1}{200}$ unit, and an elephant is a sphere with a radius of 5 units.

1. The *raw strength* of a living creature is the cross-sectional area of its muscles. The cross section of each of our spherical beings is a circle of radius r where r is the creature's radius. Order the human, ant, and elephant by their *raw strength* from least to greatest. Show your reasoning.

2. *Relative strength* is given by the ratio of raw strength to volume. It measures how strong a creature is for its size. Create an expression for the relative strength of a spherical being with radius r. (Remember that the raw strength formula is πr^2 and the volume formula for a sphere is $\frac{4}{3}\pi r^3$.)

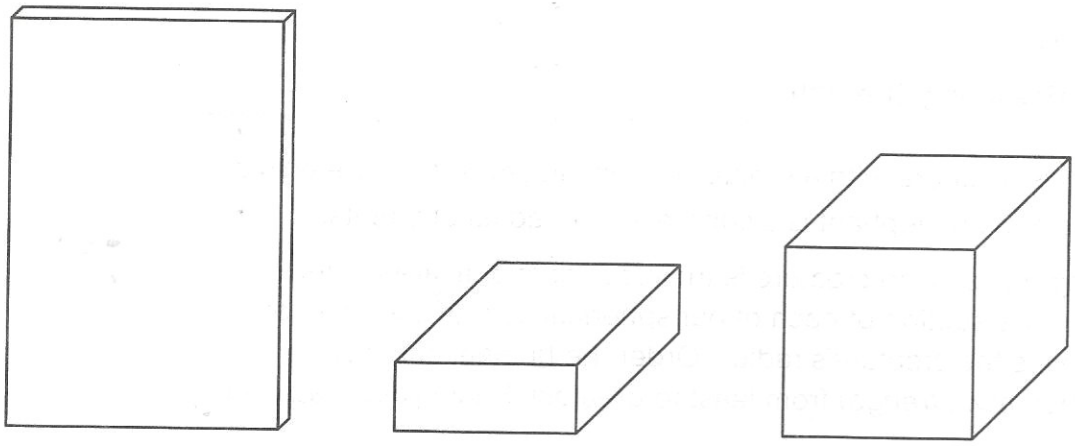
3. Order the human, the ant, and the elephant by their *relative strength*. Which is the strongest for its size?

Summary

Surface Area and Volume

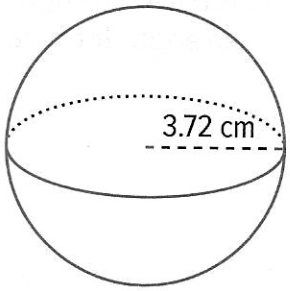
The three prisms shown each have a volume of 216 cubic centimeters. Which prism do you think has the largest surface area? Which do you think has the smallest surface area?

1 cm by 12 cm by 18 cm **9 cm by 8 cm by 3 cm** **6 cm by 6 cm by 6 cm**



The surface area of the first prism is 492 square centimeters, the surface area of the second is 246 square centimeters, and the surface area of the third is 216 square centimeters. The cube, then, has the smallest surface area. In general, the cube is the rectangular prism with the least amount of surface area for its volume. The 1 by 12 by 18 cm prism has the greatest surface area of the three. If we want a rectangular prism to have more surface area, the best design is to make it wide and long.

Shapes that are more compact, like a cube, have the least surface area for a given volume. But it turns out spheres do even better than cubes. A sphere with radius 3.72 centimeters, shown in the figure, has an approximate volume of 216 cubic centimeters like the earlier prisms, but its approximate surface area is just 174 square centimeters.



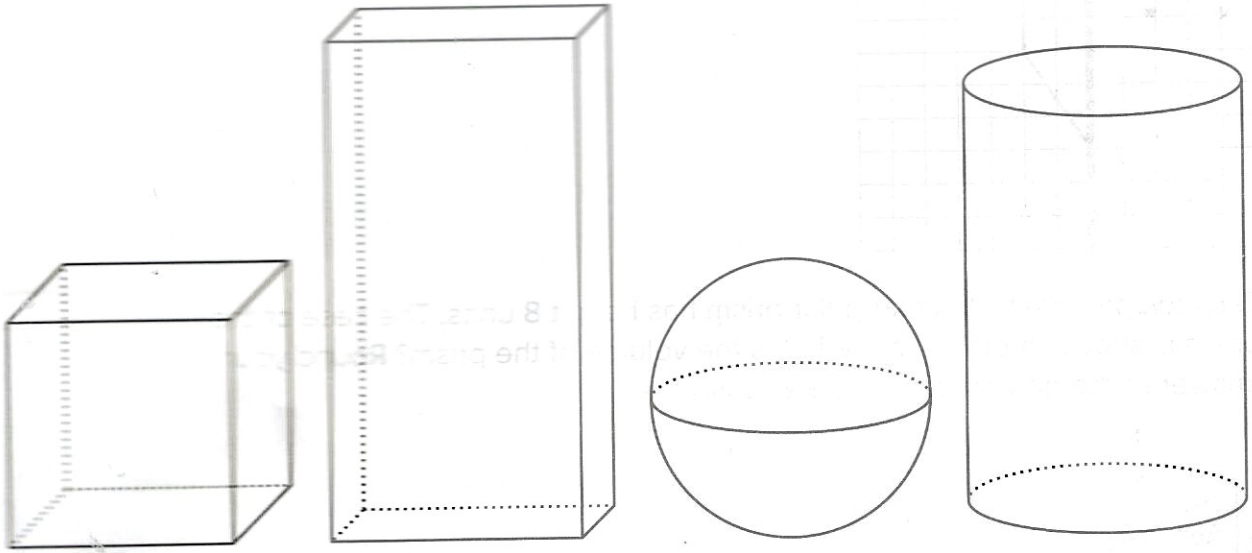
We can see examples of maximizing or minimizing surface area in nature. For instance, a snake is cold-blooded, meaning that it gets its heat from the environment. Its long, narrow shape helps it soak up more heat from the sun. On the other hand, large mammals such as elephants and cows are warm-blooded, which means that they produce their own heat internally. Their shapes are more compact, closer to spheres, and this allows them to lose as little heat as possible through their skin.

NAME _____ DATE _____ PERIOD _____

Practice

Surface Area and Volume

1. These solids all have the same volume. Which has the least surface area?



(A) Solid A (B.) Solid B (C.) Solid C (D.) Solid D

2. Rectangular prism *A* measures 3 inches by 4 inches by 8 inches. Rectangular prism *B* measures 5 inches by 5 inches by 6 inches.

   a. Before doing any calculations, predict which prism has greater surface area to volume ratio.

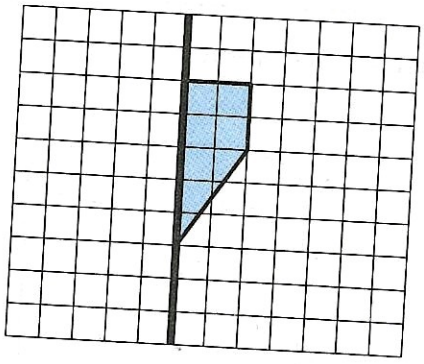
   b. Calculate the surface area, volume, and surface area to volume ratio for each prism.

3. Suppose you have 2 pieces of ice with the same volume but in different shapes. If one of the pieces has a greater surface area than the other, it will cool a beverage faster than the ice with less surface area.

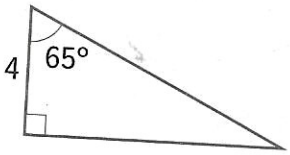
   a. Describe 2 different pieces of ice that have the same volume, but have different surface areas.

   b. Which piece of ice will cool a beverage faster?

4. Suppose this two-dimensional figure is rotated 360 degrees using the vertical axis shown. Each small square on the grid represents 1 square inch. What is the volume of the three-dimensional figure? (Lesson 5-15)



5. *Technology required.* A triangular prism has height 8 units. The base of the prism is shown in the image. What is the volume of the prism? Round your answer to the nearest tenth. (Lesson 5-15)



6. A cone-shaped container is oriented with its circular base on the top and its apex at the bottom. It has a radius of 12 inches and a height of 8 inches. The cone starts filling up with water. What fraction of the volume of the cone is filled when the water reaches a height of 2 inches? (Lesson 5-14)

7. Find the volume of a pyramid whose base is a square with sides of length 4 units and which has a height of 10 units. (Lesson 5-13)

8. A solid has volume 4 cubic units and surface area 10 square units. The solid is dilated, and the image has volume 108 cubic units. What is the surface area of the image? (Lesson 5-8)

Lesson 5-17

Volume and Density

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's use volume and density to solve problems.

Warm Up

17.1 A Kilogram by Any Other Name

Which has more mass, a thousand kilograms of feathers or a thousand kilograms of steel? Explain your reasoning.

© Holly Hildreth/McGraw Hill Education, (t) James Hardy/PhotoAlto

Activity

17.2 Light as a Feather

The feathers in a pillow have a total mass of 59 grams. The pillow is in the shape of a rectangular prism measuring 51 cm by 66 cm by 7 cm.

A steel anchor is shaped like a square pyramid. Each side of the base measures 20 cm, and its height is 28 cm. The anchor's mass is 30 kg.

1. What's the **density** of feathers in kilograms per cubic meter?

2. What's the density of steel in kilograms per cubic meter?

3. What's the volume of 1,000 kg of feathers in cubic meters?

4. What's the volume of 1,000 kg of steel in cubic meters?

Are you ready for more?

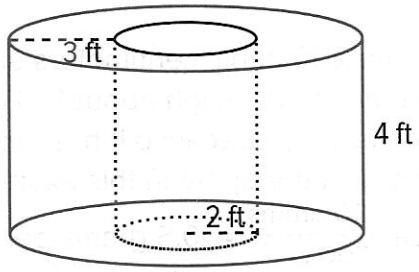
Iridium is one of the densest metals. How many times heavier would a standard pencil be if it were made out of iridium instead of wood?

NAME _____ DATE _____ PERIOD _____

Activity

17.3 A Fishy Situation

An aquarium manager drew a blueprint for a cylindrical fish tank. The tank has a vertical tube in the middle in which visitors can stand and view the fish.



The best average density for the species of fish that will go in the tank is 16 fish per 100 gallons of water. This provides enough room for the fish to swim while making sure that there are plenty of fish for people to see.

The aquarium has 275 fish available to put in the tank. Is this the right number of fish for the tank? If not, how many fish should be added or removed? Explain your reasoning.

Summary

Volume and Density

Imagine you have a baseball and an apple the size of a baseball. If we weigh each, we'll likely find that even though they're the same size, the baseball weighs more.

A baseball has volume 200 cubic centimeters and weighs 145 grams, while an apple the same volume might weigh about 100 grams. We say that the baseball is more *dense* than the apple because it has more mass packed into each unit of volume. The density of the apple in this example is 0.5 grams per cubic centimeter, because $\frac{100 \text{ grams}}{200 \text{ cm}^3} = 0.5$ grams per cubic centimeter. For the baseball, the density density is $\frac{145 \text{ grams}}{200 \text{ cm}^3} = 0.725$ grams per cubic centimeter.

In general, to find the **density** of an object, divide its mass by its volume.

Glossary

density

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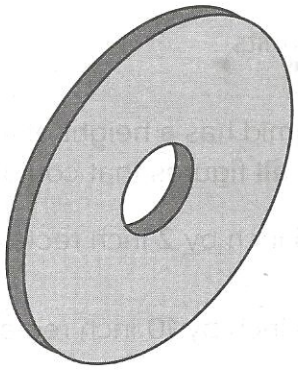
Practice

Volume and Density

1. The washer in the image has an inner diameter of $\frac{1}{4}$ inch. The outer diameter measures $\frac{3}{4}$ inch, and the washer is $\frac{1}{4}$ inch thick. The density of the metal the washers are made of is 0.285 pounds per cubic inch. How much do 5 washers weigh, in pounds? Round your answer to the nearest hundredth.



2. Assume that a cell is a sphere with radius 10^{-3} or 0.001 centimeter, and that a cell's density is 1.1 grams per cubic centimeter.

   a. Koalas weigh 6 kilograms on average. How many cells are in the average koala?

   b. Hippos weigh 1,400 kilograms on average. How many cells are in the average hippo?

3. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a toy in the shape of a rectangular prism that is 1 centimeter by 2 centimeter by 2 centimeter with mass 3 grams sink or float? Explain your reasoning.

4. A cube and a sphere both have volume 512 cubic units. Which solid has a greater surface area? Explain your reasoning. (Lesson 5-16)

5. Give the dimensions of 2 solids with equal surface area and different volume. (Lesson 5-16)

6. A right cone has a base with diameter 10 units. The volume of the cone is 100π cubic units. What is the length of a segment drawn from the apex to the edge of the circular base? (Lesson 5-15)

(A.) 5 units

(B.) 12 units

(C.) 13 units

(D.) 15 units

7. A pyramid has a height of 4 inches and a volume of 40 cubic inches. Select **all** figures that could be the base for this pyramid. (Lesson 5-14)

(A.) a 5 inch by 2 inch rectangle

(B.) a 3 inch by 10 inch rectangle

(C.) a triangle with height 10 inches and base 3 inches

(D.) a right triangle with one side 5 inches and the hypotenuse 13 inches

(E.) a heart with area 30 square inches

8. Select **all** solids for which the formula $V = Bh$ applies. (Lesson 5-9)

(A.) a triangular prism

(B.) a triangular pyramid

(C.) a square pyramid

(D.) a rectangular prism

(E.) a cone

(F.) a cylinder

9. Two distinct lines, ℓ and m, are each perpendicular to the same line n. A fourth distinct line, k, is also perpendicular to line n. Does line k intersect line ℓ or line m? Explain how you know. (Lesson 1-6)

Lesson 5-18

Volume and Graphing

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Learning Goal Let's use volume and graphing to solve problems.

Warm Up

18.1 Parade Balloon: Part 1

A company makes giant balloons for parades. They're designing a balloon that will be a dilated version of a drum similar to the one in the image. The real-life drum's diameter is 36 inches and it's 1 foot wide.

Jupiterimages/liquidlibrary/Getty Images

1. What's the approximate volume of the real-life drum in cubic feet? Round to the nearest hundredth.

2. Suppose the drum is dilated by scale factor k. Write an equation that gives the volume, V, of the dilated drum.

3. What are some reasons the actual drum volume might be different from what you calculated?

Activity

18.2 Parade Balloon: Part 2

A company makes giant balloons for parades. They're designing a balloon that will be a dilated version of a drum with diameter 36 inches and height 1 foot. The balloon will be inflated with a gas called helium. The balloon designers want to be able to find the scale factor they can achieve with different volumes of helium.

In an earlier activity, you found the volume of the original drum, and you wrote an equation to describe the volume, V, of a version of the drum that had been dilated by a factor of k.

1. Rearrange your equation to solve for k.

2. Use graphing technology to graph your rearranged equation. Set the viewing window to show a maximum of about 15,000 cubic feet on the x-axis.

3. The point (1,000, 5.21) is on the graph. What does this point mean?

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4. One tank of helium contains enough gas to fill 500 cubic feet of space. Suppose the company can afford 12 tanks.

   a. What scale factor can they use?

   b. What will the diameter of the balloon be?

5. The company received a donation that will double the number of helium tanks they can afford. How will this change the diameter of the balloon they can create?

6. The company learned that the parade route has size restrictions. The balloon can be no more than 15 feet in diameter.

   a. What scale factor would they need to get this diameter?

   b. What would the volume of the balloon be in this case?

   c. How many helium tanks would be required?

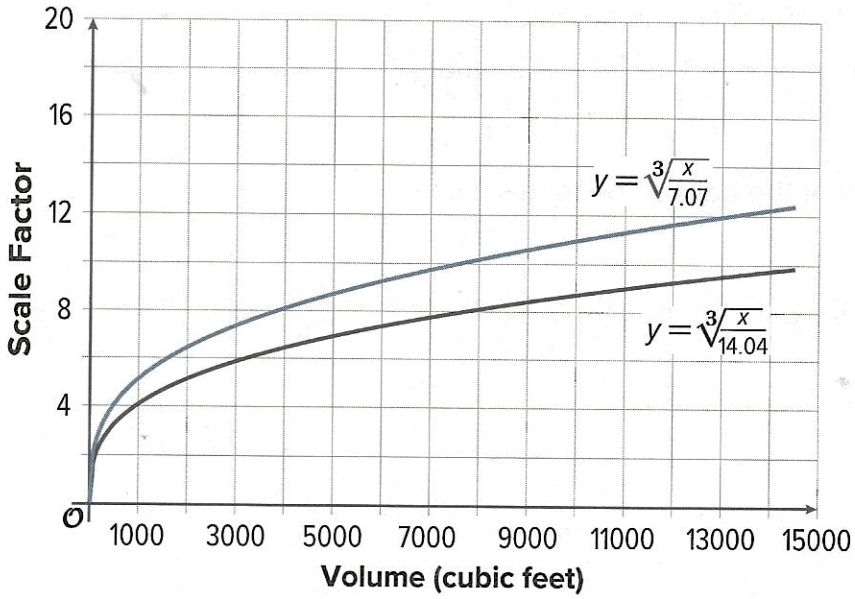
Activity

18.3 Beach Ball Balloon

The parade balloon company will make a second balloon in the shape of a beach ball that has radius 1.5 feet. The volume of the original beach ball is about 14.14 cubic feet.

1. Explain why the equation $k = \sqrt[3]{\frac{V}{14.14}}$ gives the scale factor k needed to achieve a dilated volume of V.

2. Two equations are graphed that show the relationship between volume and scale factor for the drum and the beach ball: $k = \sqrt[3]{\frac{V}{7.07}}$ and $k = \sqrt[3]{\frac{V}{14.14}}$. Use a ruler to draw a horizontal line on the graph at $k = 5$.



What do points on this line represent?

3. The company wants the two balloons to have the same scale factor. If the scale factor is 5, how much helium will they use in total?

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4. Suppose the company purchases a total of 12,000 cubic feet of helium.
   a. About what scale factor should they use if they want to use all their helium and have the same scale factor for each balloon?

   b. What will be the approximate radius of the scaled beach ball in this case?

Are you ready for more?

1. Write an expression that gives the ratio of surface area to volume of a sphere with radius r.

2. Create a graph with the sphere's radius, r, on the x-axis and the ratio you wrote on the y-axis.

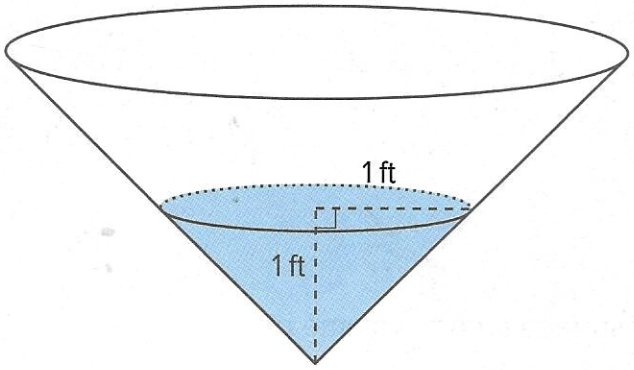
3. Describe the shape of the graph for small radii.

4. Describe the shape of the graph for large radii.

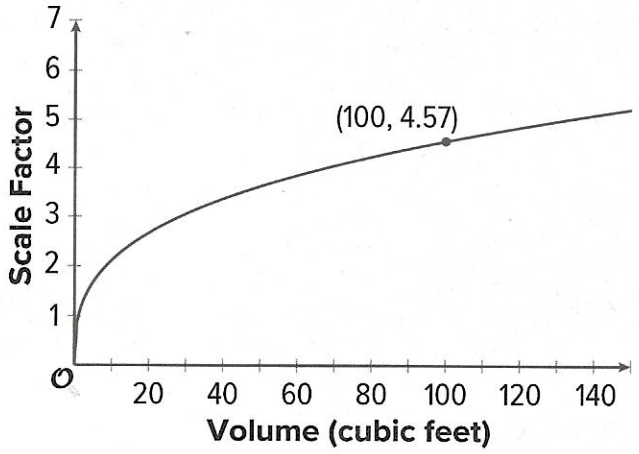
Summary

Volume and Graphing

Suppose a farm has a water tank shaped like a cone. Water is poured in from the top, and a valve can be opened to let the water flow out the bottom. There is currently a small volume of water in the tank. The section of the cone that is filled has radius 1 foot and height 1 foot.



Using the expression $\frac{1}{3}\pi r^2 h$, we find that the volume of water in the tank is about 1.05 cubic feet because $\frac{1}{3}\pi(1)^2(1) \approx 1.05$. As more water is poured into the tank, the shape of the water will be a dilation of the original small cone. An equation that expresses the volume, V, in terms of the scale factor of dilation, k, is $V = 1.05k^3$. This equation can be rearranged, resulting in $k = \sqrt[3]{\frac{V}{1.05}}$. Here is a graph of the rearranged equation.



The point (100, 4.57) on the graph tells us that if the farmer puts 100 cubic feet of water in the tank, the scale factor of dilation will be 4.57. That means that the height of the water would be the original height of 1 foot times the scale factor 4.57, or 4.57 feet.

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Practice

Volume and Graphing

1. A cube with side length 5 centimeters has a density of 3 grams per cubic centimeter. What is its mass? (Lesson 5-17)

2. Rectangular prism *A* measures 5 inches by 5 inches by 6 inches. Rectangular prism *B* measures 2 inches by 4 inches by 6 inches. (Lesson 5-16)

   a. Before doing any calculations, predict which prism has greater surface area to volume ratio.

   b. Calculate the surface area, volume, and surface area to volume ratio for each prism.

3. A right cone has a base with diameter 4 units. The volume of the cone is 16π cubic units. What is the length of a segment drawn from the apex to the edge of the circular base? (Lesson 5-15)

4. A right pyramid has a square base with sides of length 10 units. Each segment connecting the apex to a midpoint of a side of the base has length 13 units. What is the volume of the pyramid? (Lesson 5-15)

   (A.) 1300 cubic units

   (B.) 1200 cubic units

   (C.) $\frac{1300}{3}$ cubic units

   (D.) 400 cubic units

5. A solid can be constructed with 2 squares and 4 congruent, non-rectangular parallelograms. What is the name of this solid? (Lesson 5-12)

   (A.) cube

   (B.) right rectangular prism

   (C.) right square prism

   (D.) oblique square prism

6. Diego is deciding which of 2 juice containers he should buy. One container is in the shape of a cylinder with radius 2.5 centimeters and height 10.5 centimeters. The second container is in the shape of a rectangular prism. The prism also has height 10.5 centimeters. Its length is 4 centimeters and its width is 6 centimeters. (Lesson 5-10)

   Which juice container has the larger volume?

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7. A parallelogram has an area of 1 square centimeter. Write an equation where y is the scale factor required for a dilation of the parallelogram to have an area of x square units. Sketch a graph representing the equation. (Lesson 5-5)

8. Suppose several solids are divided into thin slices, all in the same direction. For each set of slices, decide what kind of solid they came from. (Lesson 5-2)

   a. a set of similar rectangles, decreasing in size to a single point, ordered from greatest in size to smallest

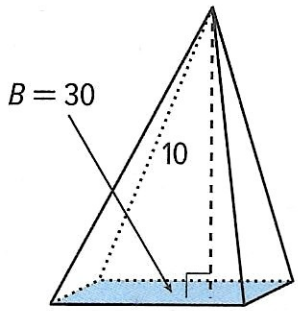
   b. a set of congruent triangles

   c. a set of congruent squares

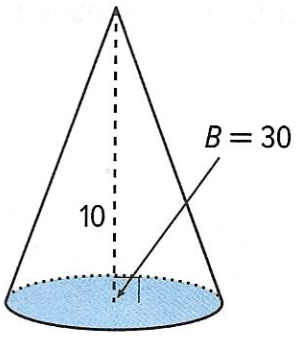
   d. a set of circles, decreasing in size to a single point, ordered from greatest in size to smallest

9. Four solids on the list have the same volume. Select these solids. (Lesson 5-13)

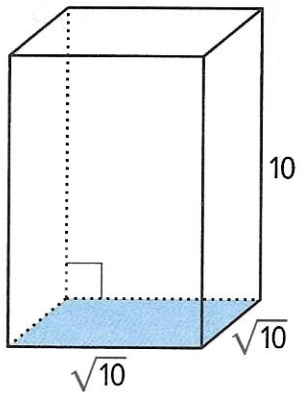
Solid A



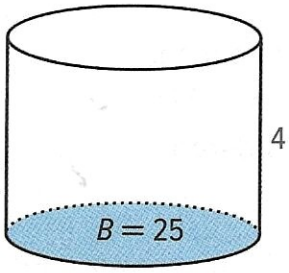
Solid B



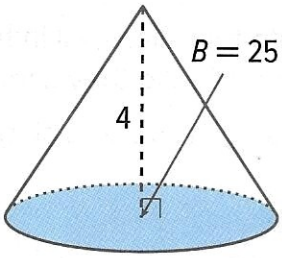
Solid C



Solid D



Solid E



(A.) Solid A

(B.) Solid B

(C.) Solid C

(D.) Solid D

(E.) Solid E

Learning Targets

Lesson	Learning Target(s)
5-1 Solids of Rotation	• I can draw the two-dimensional shape that creates a particular three-dimensional solid when rotated using a given axis. • I can identify the three-dimensional solid created by rotating a two-dimensional figure using a linear axis.
5-2 Slicing Solids	• I can identify the three-dimensional shape that generates a set of cross sections. • I can visualize and draw multiple cross sections of a three-dimensional figure.
5-3 Creating Cross Sections by Dilating	• I know that a pyramid's cross sections are dilations of its base with scale factors ranging from 0 to 1.
5-4 Scaling and Area	• I know that when figures are dilated by a scale factor of k, their areas are multiplied by k^2.

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Lesson	Learning Target(s)
5-5 Scaling and Unscaling	• I can use square root graphs and do calculations to interpret the relationships between scale factors and areas.
5-6 Scaling Solids	• I know that when a solid is dilated by a scale factor of k, its surface area is multiplied by k^2 and its volume is multiplied by k^3.
5-7 The Root of the Problem	• I can create and describe graphs that show relationships between volumes and scale factors. • I can work backwards from a volume or surface area scaling to find a scale factor.
5-8 Speaking of Scaling	• I can calculate scale factors for lengths, surface areas, and volumes if I'm given any 1 of the 3 factors.
5-9 Cylinder Volumes	• I can calculate volumes of solids that are composed of cylinders. • I can explain how finding the volume of a prism relates to finding the volume of a cylinder.

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Lesson	Learning Target(s)
5-10 Cross Sections and Volume	• I know that if two solids have equal-area cross sections at all heights, they have the same volumes.
5-11 Prisms Practice	• I can calculate volumes of right and oblique prisms and cylinders and figures composed of prisms and cylinders.
5-12 Prisms and Pyramids	• I can explain the relationships between pyramids, cones, prisms, and cylinders.
5-13 Building a Volume Formula for a Pyramid	• I can explain why the volume formula for pyramids and cones is $V = \frac{1}{3}Bh$.
5-14 Working with Pyramids	• I can calculate volumes of pyramids and cones. • I can work backward from a given volume to find possible dimensions of a pyramid or cone.

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Lesson	Learning Target(s)
5-15 Putting All the Solids Together	• I can use the Pythagorean Theorem and trigonometry to help calculate volumes of prisms, cylinders, cones, and pyramids, including solids of rotation.
5-16 Surface Area and Volume	• I can use surface area and volume relationships to solve problems.
5-17 Volume and Density	• I can solve problems involving density and volume. • I know that the density of an object is the ratio between its mass and its volume.
5-18 Volume and Graphing	• I can use cube root and square root graphs to solve geometric problems.

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